Chapter 4 Rational Functions and Conics

Course/Section
Lesson Number
Date

Section 4.5 Translations of Conics

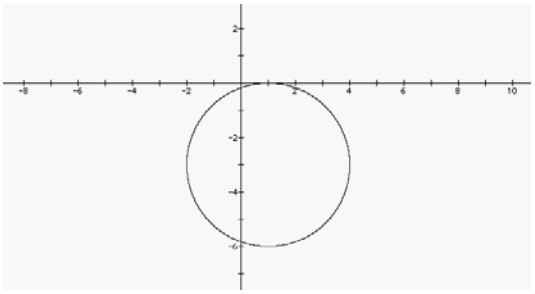
Section Objectives: Students will know how to recognize, graph, and write equations of conics that have been shifted vertically or horizontally in the plane.

I. Vertical and Horizontal Shifts of Conics (pp. 371-372) Pace: 10 minutes

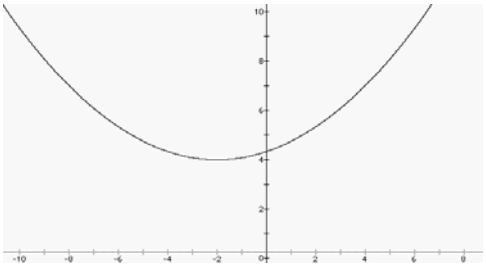
- State that the only new idea in this section is that the center (or vertex for a parabola) is no longer at (0, 0), it is at (h, k).

Example 1. Sketch the graphs of the following conics.

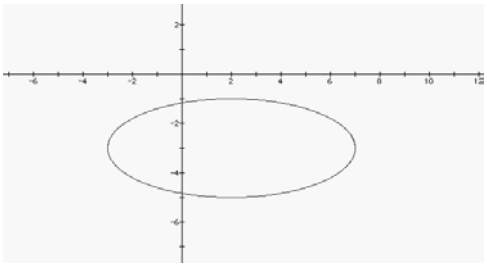
a) $(x-1)^2 + (y+3)^2 = 9$.



b) $(x+2)^2 = 12(y-4)$



c) $\dfrac{(x-2)^2}{25} + \dfrac{(y+3)^2}{4} = 1$



II. Equations of Conics in Standard Form (pp. 373-375) Pace: 15 minutes

- State these facts, which come from the previous section.
  1. **Parabola:** p is the directed distance from the vertex to the focus.

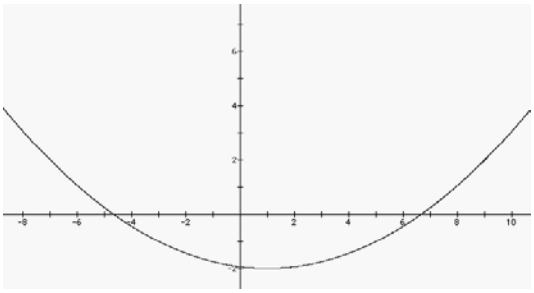
  2. **Ellipse:** a is the distance from the center to the vertices, b is the distance from the center to the covertices, and c is the distance from the center to the foci.

  3. **Hyperbola:** a is the distance from the center to the vertices and c is the distance from the center to the foci.

Example 2. Find the vertex, focus, and directrix of the parabola given by

$$x^2 - 2x - 16y - 31 = 0$$
$$x^2 - 2x = 16y + 31$$
$$x^2 - 2x + 1 = 16y + 31 + 1$$
$$(x-1)^2 = 16(y+2)$$

So, the vertex is at (1, -2) and $p = 4$. That gives us a focus at (1, 2), and the directrix is $y = -6$.



Example 3. Sketch the graph of the following ellipse.

$$25x^2 + 9y^2 - 200x + 36y + 211 = 0$$

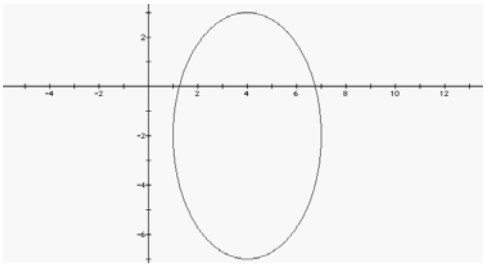
$$25\left(x^2 - 8x\right) + 9\left(y^2 + 4y\right) = -211$$

$$25\left(x^2 - 8x + 16\right) + 9\left(y^2 + 4y + 4\right) = -211 + 400 + 36$$

$$25(x-4)^2 + 9(y+2)^2 = 225$$

$$\frac{(x-4)^2}{9} + \frac{(y+2)^2}{25} = 1$$

So, the center is at (4, -2), $a = 5$, and $b = 3$. To plot the vertices go up and down 5 units from the center. To plot the covertices go right and left 3 units from the center.



Example 4. Sketch the graph of the hyperbola given by

$$4x^2 - 9y^2 - 24x - 72y - 72 = 0$$

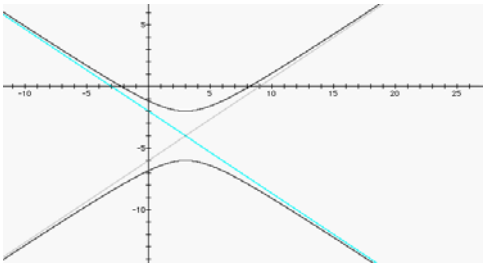
$$4\left(x^2 - 6x\right) - 9\left(y^2 + 8y\right) = 72$$

$$4\left(x^2 - 6x + 9\right) - 9\left(y^2 + 8y + 16\right) = 72 + 36 - 144$$

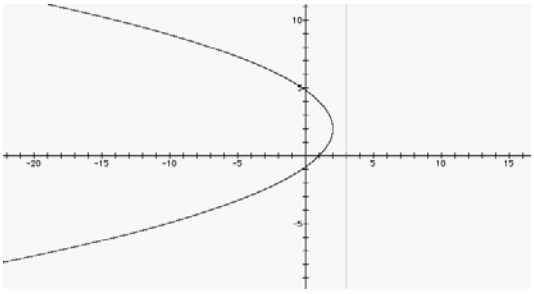
$$4(x-3)^2 - 9(y+4)^2 = -36$$

$$\frac{(y+4)^2}{4} - \frac{(x-3)^2}{9} = 1$$

So, the center is at (3, -4), $a = 2$, and $b = 3$. Go up and down 2 units from the center and right and left 3 units from the center. Use these four points to form the box and draw the asymptotes.



Example 5. Write the standard form equation of the parabola with focus at (1, 2) and directrix $x = 3$.
The vertex will be at (2, 2) and $p = -1$. Hence the equation is $(y - 2)^2 = -4(x - 2)$.



- Assign the *Writing About Mathematics* on page 375 of the text.