MS 101 **Sample Test 2** (Section 5.5)

$A = P e^{r t}$ $A = P(1 + \frac{r}{n})^{n t}$ $P = P_0 e^{k t}$ $Q = Q_0 e^{k t}$

Solutions can be found at the bottom of this file.

01. (06 pts) $1250 is invested at an APR of 6% compounded continuously.

a) What is the value after 15 years?

b) How long will it take for the investment to double in value?

02. (06 pts) $1300 is invested and compounded continuously. After 13 years the investment has doubled. Find the value r (4 decimals) in the model and express it as an APR(*annual percentage rate*).

r = ________ **APR** = ________

03. (04 pts) The population of Shanty Town can be modeled by $P = 1500\, e^{.017 t}$ where $t = 0$ represents year 2000 and P is the number of people. What will the population be in the year 2007?

POPULATION = __________ (round to the nearest whole number)

04. (04 pts) The population of Shankstown can be modeled by $P = 1500\, e^{k t}$ where $t = 0$ represents year 1990. In 2005 the population was 1700 people. Find k (the continuous growth rate).

k = ___________
(4 decimals)

05. (06 pts) The number of bacteria in a petri dish is modeled by $N = N_0 e^{kt}$ where t is in days and N is the number of bacteria. On day 2 there are 2,000 bacteria and on day 7 there are 12,000 bacteria.
Find N_0 and k and write the model as an equation.

k = ___________ ***N***$_0$ = __________ **THE MODEL:** _____________________ (This is an equation.)
(4 decimals) (2 decimals)

06. (04 pts) A radioactive isotope decays according to $Q = 14\, e^{kt}$ and has a half-life of 2,707 years. Find the vaue of k.

k = ___________
(6 decimals)

07. (04 pts) What is the half-life of a decaying substance if the initial amount present is 9 grams and there are 2 grams left after 19,000 years:

half-life = ___________

08. (06 pts) **Depreciation.** Suppose you buy a new car. The car's value decreases according to $V = 15000\, e^{-.0375\, t}$ where V value of the car(in dollars) and t is the age of the car (in years).

a) What the price you paid for the car?

b) Suppose you want to sell the car when it value has decreased to $9,000. What will be the age of the car then?

================================ SOLUTIONS ARE BELOW= ==

01. (06 pts) $1250 is invested at an APR of 6% compounded continuously.

a) What is the value after 15 years? $1250\, e^{(.06*15)} = \$3,074.50$

b) How long will it take for the investment to double in value?

$2500 = 1250\, e^{(.06*t)}$

$\frac{2500}{1250} = \frac{1250\, e^{(.06*t)}}{1250}$

$2 = e^{(.06*t)}$

$\ln(2) = \ln(e^{(.06*t)})$

$\ln(2) = .06 * t$

$\frac{\ln(2)}{.06} = t \approx 11.55 \text{ years}$

02. (06 pts) \$1300 is invested and compounded continuously. After 13 years the investment has doubled.
Find the value r (4 decimals) in the model and express it as an APR(*annual percentage rate*).

r = __0.0533_____ **APR** = __ 5.33%_____

$2600 = 1300\, e^{(r*13)}$

$\frac{2600}{1300} = \frac{1300\, e^{(r*13)}}{1300}$

$2 = e^{(r*13)}$

$\ln(2) = \ln(e^{(r*13)})$

$\ln(2) = r * 13$

$\frac{\ln(2)}{13} = r \approx 0.0533 = 5.33\,\%$

03. (04 pts) The population of Shanty Town can be modeled by $P = 1500\, e^{.017\, t}$ where $t = 0$ represents year 2000 and P is the number of people. What will the population be in the year 2007?

POPULATION = __1690 people_______ (round to the nearest whole number) $P = 1500\, e^{.017*7} \approx 1690$ people

04. (04 pts) The population of Shankstown can be modeled by $P = 1500\, e^{k\, t}$ where $t = 0$ represents year 1990. In 2005 the population was 1700 people. Find k (the continuous growth rate).

k = ___0.0083_______
(4 decimals)

$1700 = 1500\, e^{k*15}$

$\frac{1700}{1500} = \frac{1500\, e^{k*15}}{1500}$

$\frac{1700}{1500} = e^{k*15}$

$\ln\left(\frac{1700}{1500}\right) = \ln(e^{k*15})$

$\ln\left(\frac{1700}{1500}\right) = k * 15$

$\frac{\ln\left(\frac{1700}{1500}\right)}{15} = k \approx 0.0083$

05. (06 pts) The number of bacteria in a petri dish is modeled by $N = N_0\, e^{k t}$ where t is in days and N is the number of bacteria. On day 2 there are 2,000 bacteria and on day 7 there are 12,000 bacteria.
Find N_0 and k and write the model as an equation.

k = ____0.3584______ **N_0** = __976.39_______ **THE MODEL:** ____ $N = 976.39\, e^{0.3584\, t}$ ___ (This is an equation.)

(4 decimals) (2 decimals)

You can write two equations with the two given data points:

(2,2000) (7,12000)

$2000 = N_0\, e^{k \cdot 2}$ and $12000 = N_0\, e^{k \cdot 7}$

Divide the equations:

$\frac{12000}{2000} = \frac{N_0\, e^{k \cdot 7}}{N_0\, e^{k \cdot 2}}$

$\Longrightarrow \frac{12000}{2000} = \frac{e^{k \cdot 7}}{e^{k \cdot 2}} \Longrightarrow \frac{12000}{2000} = e^{k \cdot 5} \Longrightarrow \ln\left(\frac{12000}{2000}\right) = \ln(e^{5\, k}) \Longrightarrow \ln\left(\frac{12000}{2000}\right) = 5\, k \Longrightarrow \frac{\ln\left(\frac{12000}{2000}\right)}{5} = k \approx 0.3584$

Find N_0: $12000 = N_0\, e^{0.3584*7}$

$\frac{12000}{e^{0.3584*7}} = N_0 \approx 976.39$

06. (04 pts) A radioactive isotope decays according to $Q = 14\, e^{k t}$ and has a half-life of 2,707 years. Find the vaue of k.

k = ___ -0.000256 ________

(6 decimals)

A half-life of 2,707 years means $Q = 7$ when $t = 2707$.

$7 = 14\, e^{k\,2707}$

$\frac{7}{14} = \frac{14\, e^{k\,2707}}{14}$

$\frac{1}{2} = e^{k\,2707}$

$\ln\left(\frac{1}{2}\right) = \ln\left(e^{k\,2707}\right)$

$\ln\left(\frac{1}{2}\right) = k\,2707$

$\frac{\ln\left(\frac{1}{2}\right)}{2707} = k \approx -0.000256$

07. (04 pts) What is the half-life of a decaying substance if the initial amount present is 9 grams and there are 2 grams left after 19,000 years:

half-life = ___8752 years________

First, you must find the decay rate, *k*:

$2 = 9\, e^{k\,19000}$

$\frac{2}{9} = \frac{9\, e^{k\,19000}}{9}$

$\frac{2}{9} = e^{k\,19000}$

$\ln\left(\frac{2}{9}\right) = \ln\left(e^{k\,19000}\right)$

$\ln\left(\frac{2}{9}\right) = k\,19000$

$\frac{\ln\left(\frac{2}{9}\right)}{19000} = k \approx -0.0000792$

$\text{Recall, the half} - \text{life} = \frac{\ln\left(\frac{1}{2}\right)}{k} = \frac{\ln\left(\frac{1}{2}\right)}{-0.0000792} = 8752\,\text{years}$

08. (06 pts) **Depreciation.** Suppose you buy a new car. The car's value decreases according to $V = 15000\, e^{-.0375\, t}$ where V value of the car(in dollars) and t is the age of the car (in years).

a) What the price you paid for the car? $15000

b) Suppose you want to sell the car when it value has decreased to $9,000. What will be the age of the car then?

Set $V = 9000$ then solve for t :

$9000 = 15000\, e^{-.0375\, t}$

$\frac{\ln\left(\frac{9000}{15000}\right)}{-.0375} = t \approx 13.62$ years