MS 101 **Sample Test 2** (5.4,5.5)

$A = P e^{rt}$ $A = P(1 + \frac{r}{n})^{nt}$ $P = P_0 e^{kt}$ $Q = Q_0 e^{kt}$

Solutions can be found at the bottom of this file.

01. (04 pts) Solve for x: $2 e^{3x} = 12$

02. (04 pts) Solve for x: $10^{3x} = 12$

03. (04 pts) Solve for x: $5 \ln(3x) = 20$

04. (04 pts) Solve for x: $5 \log(4x) = 25$

05. (06 pts) \$1250 is invested at an APR of 6% compounded continuously.

a) What is the value after 15 years?

b) How long will it take for the investment to double in value?

06. (06 pts) \$1300 is invested and compounded continuously. After 13 years the investment has doubled. Find the value r (4 decimals) in the model and express it as an APR(*annual percentage rate*).

r = ________ **APR** = ________

07. (04 pts) The population of Shanty Town can be modeled by $P = 1500 e^{.017 t}$ where $t = 0$ represents year 2000 and P is the number of people. What will the population be in the year 2007?

POPULATION = __________ (round to the nearest whole number)

08. (04 pts) The population of Shankstown can be modeled by $P = 1500\, e^{k\,t}$ where $t = 0$ represents year 1990. In 2005 the population was 1700 people. Find k (the continuous growth rate).

k = __________
(4 decimals)

09. (06 pts) The number of bacteria in a petri dish is modeled by $N = N_0\, e^{k\,t}$ where t is in days and N is the number of bacteria. On day 2 there are 2,000 bacteria and on day 7 there are 12,000 bacteria.
Find N_0 and k and write the model as an equation.

k = ___________ ***N***$_0$ = ___________ **THE MODEL:** _______________________ (This is an equation.)
(4 decimals) (2 decimals)

10. (04 pts) A radioactive isotope decays according to $Q = 14\, e^{k\,t}$ and has a half-life of 2,707 years. Find the vaue of k.

k = ___________
(6 decimals)

11. (04 pts) What is the half-life of a decaying substance if the initial amount present is 9 grams and there are 2 grams left after 19,000 years:

half-life = ___________

12. (06 pts) **Depreciation.** Suppose you buy a new car. The car's value decreases according to $V = 15000\, e^{-.0375\,t}$ where V value of the car(in dollars) and t is the age of the car (in years).

a) What the price you paid for the car?

b) Suppose you want to sell the car when it value has decreased to $9,000. What will be the age of the car then?

================================ SOLUTIONS ARE BELOW= ==

01. (04 pts) Solve for x: $2e^{3x} = 12$

$2e^{3x} = 12 \Longrightarrow e^{3x} = 6 \Longrightarrow \ln(e^{3x}) = \ln(6) \Longrightarrow 3x = \ln(6) \Longrightarrow x = \frac{\ln(6)}{3} \approx 0.597$

02. (04 pts) Solve for x: $10^{3x} = 12$

$10^{3x} = 12 \Longrightarrow \log(10^{3x}) = \log(12) \Longrightarrow 3x = \log(12) \Longrightarrow x = \frac{\log(12)}{3} \approx 0.3597$

03. (04 pts) Solve for x: $5\ln(3x) = 20$

$5\ln(3x) = 20 \Longrightarrow \ln(3x) = 4 \Longrightarrow e^{\ln(3x)} = e^4 \Longrightarrow 3x = e^4 \Longrightarrow x = \frac{e^4}{3} \approx 18.199$

04. (04 pts) Solve for x : $5\log(4x) = 25$

$5\log(4x) = 25 \Longrightarrow \log(4x) = 5 \Longrightarrow 10^{\log(4x)} = 10^5 \Longrightarrow 4x = 10^5 \Longrightarrow x = \frac{10^5}{4} = 25,000$

05. (06 pts) $1250 is invested at an APR of 6% compounded continuously.

a) What is the value after 15 years? $1250\,e^{(.06*15)} = \$3,074.50$

b) How long will it take for the investment to double in value?

$2500 = 1250\,e^{(.06*t)}$

$\frac{2500}{1250} = \frac{1250\,e^{(.06*t)}}{1250}$

$2 = e^{(.06*t)}$

$\ln(2) = \ln(e^{(.06*t)})$

$\ln(2) = .06*t$

$\frac{\ln(2)}{.06} = t \approx 11.55\text{ years}$

06. (06 pts) $1300 is invested and compounded continuously. After 13 years the investment has doubled.
Find the value r (4 decimals) in the model and express it as an APR(*annual percentage rate*).

r = __0.0533_____ **APR** = __ 5.33%_____

$2600 = 1300\, e^{(r*13)}$

$\frac{2600}{1300} = \frac{1300\, e^{(r*13)}}{1300}$

$2 = e^{(r*13)}$

$\ln(2) = \ln(e^{(r*13)})$

$\ln(2) = r*13$

$\frac{\ln(2)}{13} = r \approx 0.0533 = 5.33\,\%$

07. (04 pts) The population of Shanty Town can be modeled by $P = 1500\, e^{.017\, t}$ where $t = 0$ represents year 2000 and P is the number of people. What will the population be in the year 2007?

POPULATION = __1690 people_______ (round to the nearest whole number) $P = 1500\, e^{.017*7} \approx 1690$ people

08. (04 pts) The population of Shankstown can be modeled by $P = 1500\, e^{k\, t}$ where $t = 0$ represents year 1990. In 2005 the population was 1700 people. Find k (the continuous growth rate).

k = ___0.0083_______
(4 decimals)

$1700 = 1500\, e^{k*15}$

$\frac{1700}{1500} = \frac{1500\, e^{k*15}}{1500}$

$\frac{1700}{1500} = e^{k*15}$

$\ln\left(\frac{1700}{1500}\right) = \ln(e^{k*15})$

$\ln\left(\frac{1700}{1500}\right) = k*15$

$\frac{\ln\left(\frac{1700}{1500}\right)}{15} = k \approx 0.0083$

09. (06 pts) The number of bacteria in a petri dish is modeled by $N = N_0\, e^{k t}$ where t is in days and N is the number of bacteria. On day 2 there are 2,000 bacteria and on day 7 there are 12,000 bacteria.
Find N_0 and k and write the model as an equation.

k = ____0.3584______ (4 decimals) **N_0** = __976.39_______ (2 decimals) **THE MODEL:** ____ $N = 976.39\, e^{0.3584\, t}$ ___ (This is an equation.)

You can write two equations with the two given data points:

(2,2000) (7,12000)

$2000 = N_0\, e^{k \cdot 2}$ and $12000 = N_0\, e^{k \cdot 7}$

Divide the equations:

$$\frac{12000}{2000} = \frac{N_0\, e^{k \cdot 7}}{N_0\, e^{k \cdot 2}}$$

$$\Longrightarrow \frac{12000}{2000} = \frac{e^{k \cdot 7}}{e^{k \cdot 2}} \Longrightarrow \frac{12000}{2000} = e^{k \cdot 5} \Longrightarrow \ln\left(\frac{12000}{2000}\right) = \ln(e^{5k}) \Longrightarrow \ln\left(\frac{12000}{2000}\right) = 5k \Longrightarrow \frac{\ln\left(\frac{12000}{2000}\right)}{5} = k \approx 0.3584$$

Find N_0: $12000 = N_0\, e^{0.3584 * 7}$

$$\frac{12000}{e^{0.3584 * 7}} = N_0 \approx 976.39$$

10. (04 pts) A radioactive isotope decays according to $Q = 14\, e^{k\, t}$ and has a half-life of 2,707 years. Find the vaue of k.

k = ___ -0.000256 ________ (6 decimals)

A half-life of 2,707 years means $Q = 7$ when $t = 2707$.

$$7 = 14\, e^{k\, 2707}$$

$$\frac{7}{14} = \frac{14\, e^{k\, 2707}}{14}$$

$$\frac{1}{2} = e^{k\, 2707}$$

$$\ln\left(\frac{1}{2}\right) = \ln(e^{k\, 2707})$$

$$\ln\left(\frac{1}{2}\right) = k\, 2707$$

$$\frac{\ln\left(\frac{1}{2}\right)}{2707} = k \approx -0.000256$$

11. (04 pts) What is the half-life of a decaying substance if the initial amount present is 9 grams and there are 2 grams left after 19,000 years:

half-life = ___8752 years________

First, you must find the decay rate, *k*:

$2 = 9\,e^{k\,19000}$

$\frac{2}{9} = \frac{9\,e^{k\,19000}}{9}$

$\frac{2}{9} = e^{k\,19000}$

$\ln\left(\frac{2}{9}\right) = \ln\left(e^{k\,19000}\right)$

$\ln\left(\frac{2}{9}\right) = k\,19000$

$\frac{\ln\left(\frac{2}{9}\right)}{19000} = k \approx -0.0000792$

Recall, the half – life $= \frac{\ln\left(\frac{1}{2}\right)}{k} = \frac{\ln\left(\frac{1}{2}\right)}{-0.0000792} = 8752$ years

12. (06 pts) **Depreciation.** Suppose you buy a new car. The car's value decreases according to $V = 15000\,e^{-.0375\,t}$ where V value of the car(in dollars) and t is the age of the car (in years).

a) What the price you paid for the car? $15000

b) Suppose you want to sell the car when it value has decreased to $9,000. What will be the age of the car then?

Set $V = 9000$ then solve for t :

$9000 = 15000\,e^{-.0375\,t}$

$\frac{\ln\left(\frac{9000}{15000}\right)}{-.0375} = t \approx 13.62$ years