Section 3.1

Quadratic Functions

and

Models

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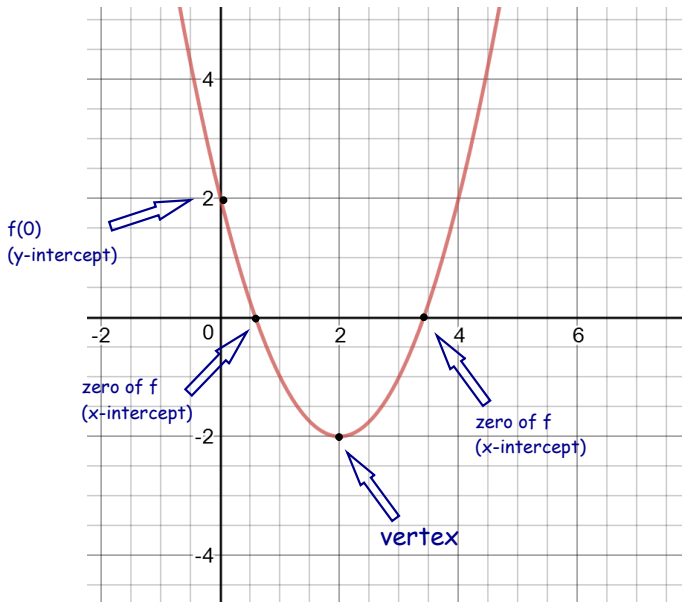
The graph of a quadratic function is called a parabola.

There will be several key points on the graph:

1. y-intercept
2. the zeros of the function
3. the vertex



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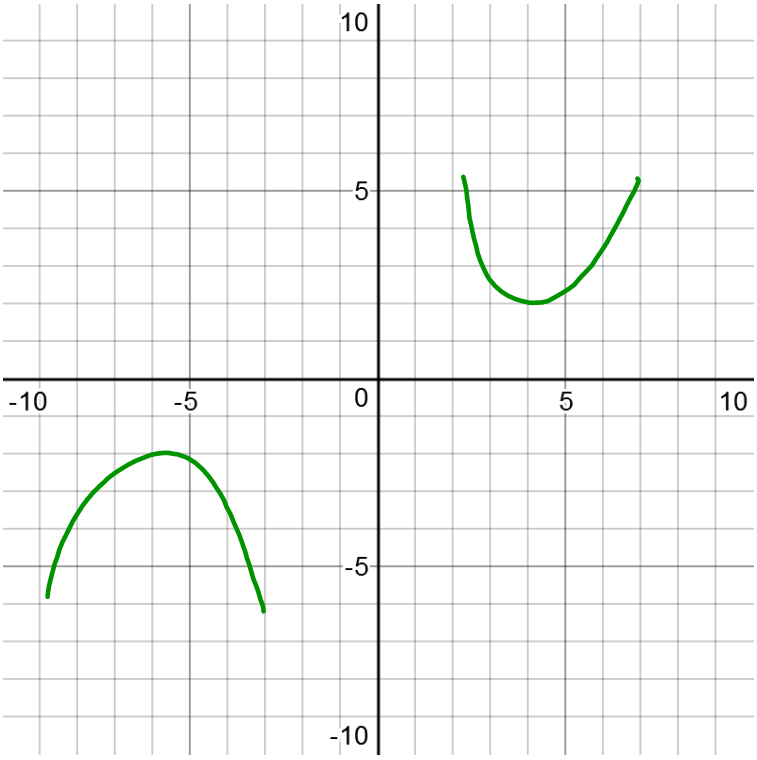
Can you draw a parabola that does not have any x-intercepts?



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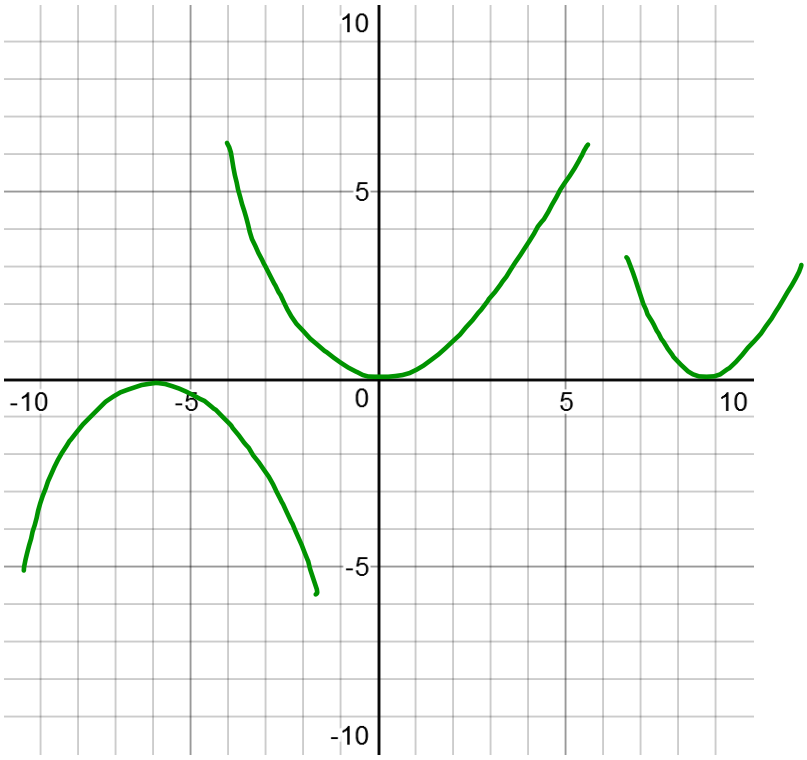
Can you draw a parabola that has exactly one x-intercept?



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The Quadratic Form of a quadratic function is:

$$f(x) = ax^2 + bx + c$$

The vertex is at the point (x,y)

$$x = \frac{-b}{2a} \qquad y = f\left(\frac{-b}{2a}\right)$$

Note: This is the same form you use when applying the Quadratic Formula.

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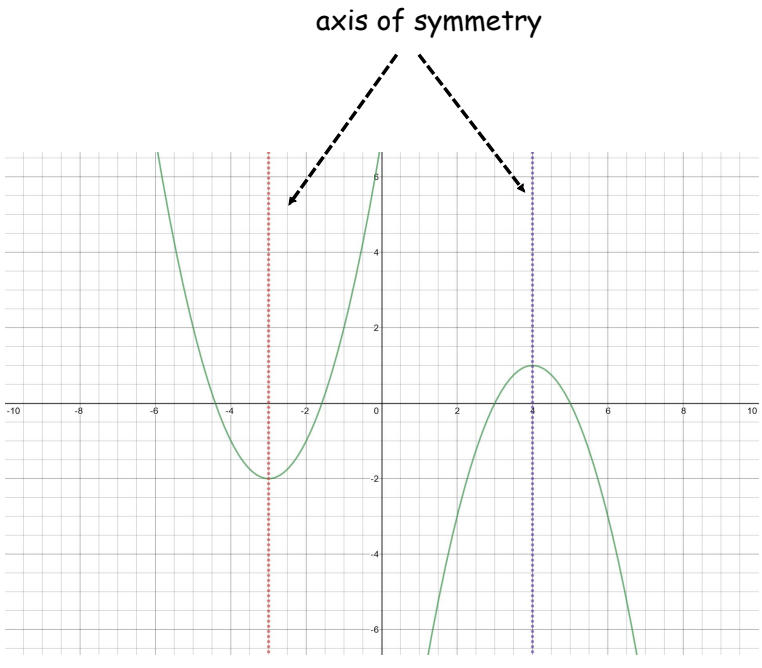
The axis of symmetry is the vertical line passing through the vertex.



Example: Finding the vertex when given the quadratic form.

$f(x) = 2x^2 - 16x + 31$

vertex is at $(x, y) = \left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$(4, -1)$

$X = \frac{-(-16)}{2(2)} = 4$

$Y = f(4) = 2(4)^2 - 16(4) + 31 = -1$

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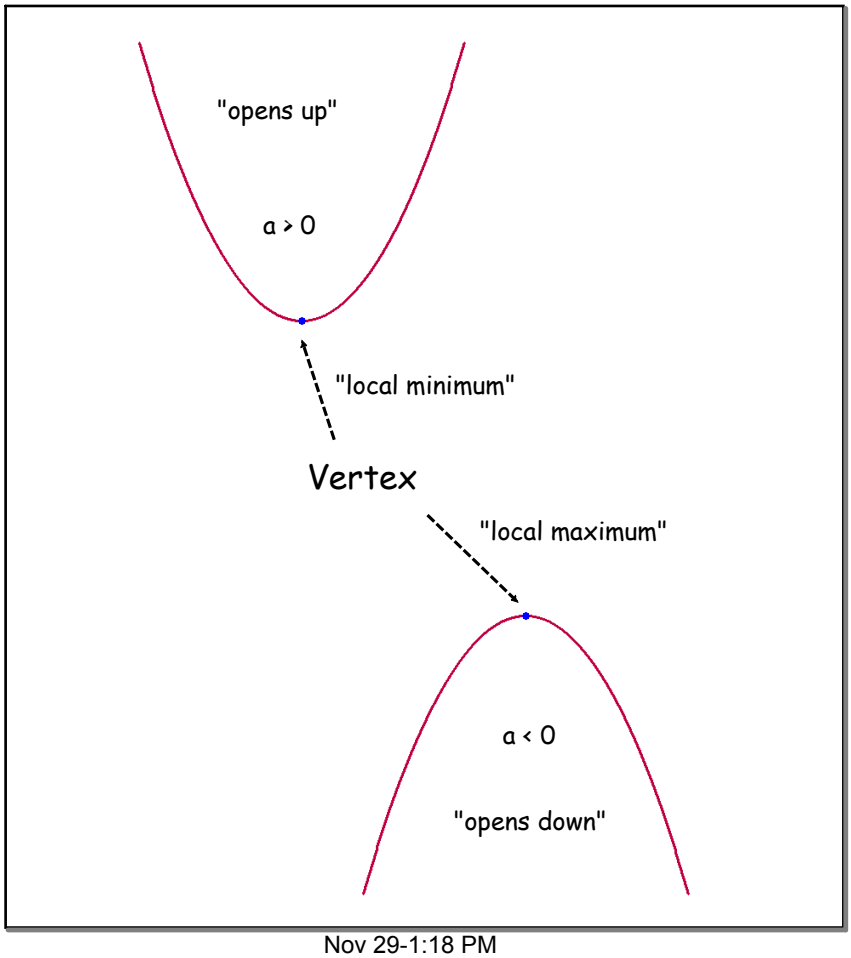
$$f(x) = ax^2 + bx + c$$

leading term

"a" is the leading coefficient



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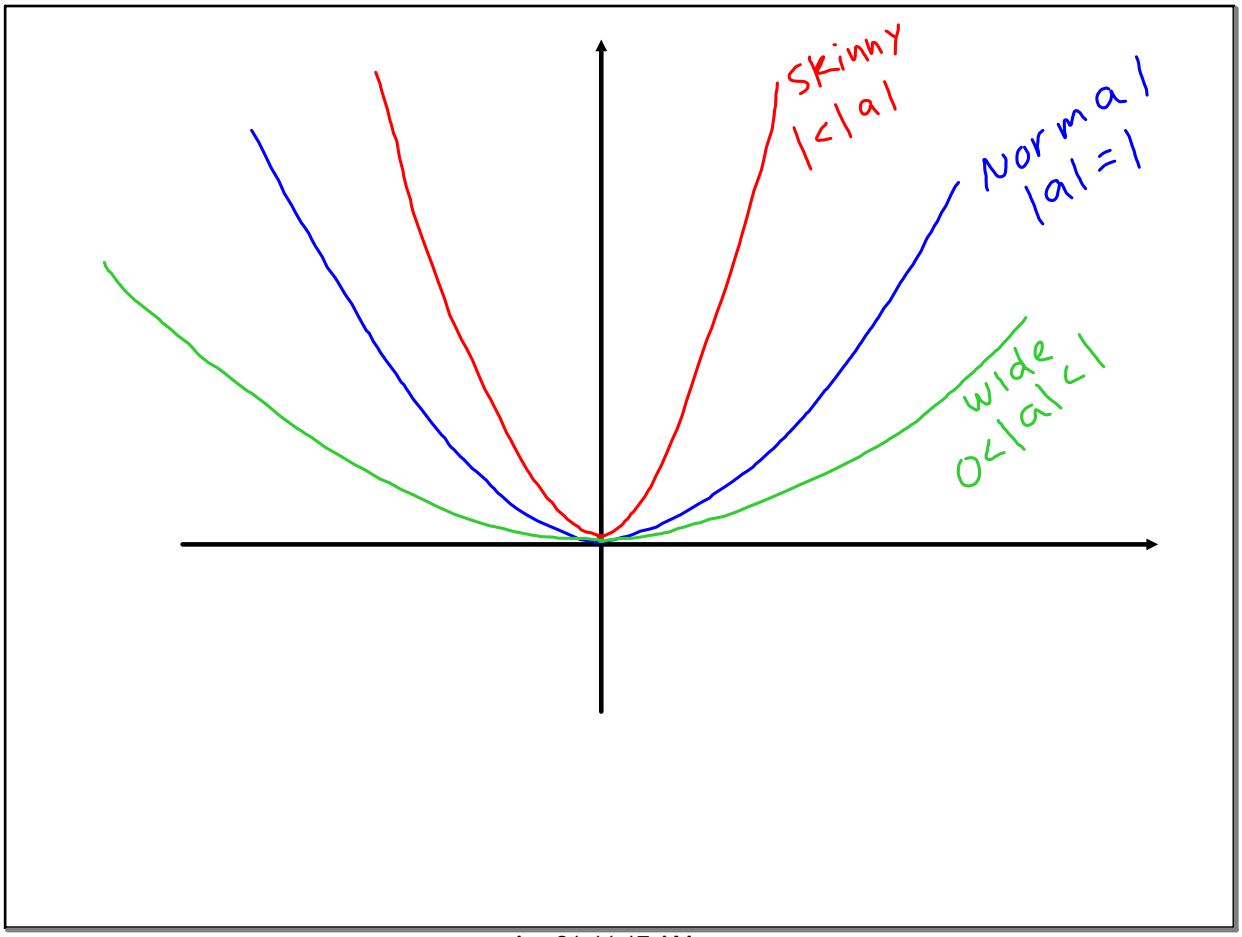

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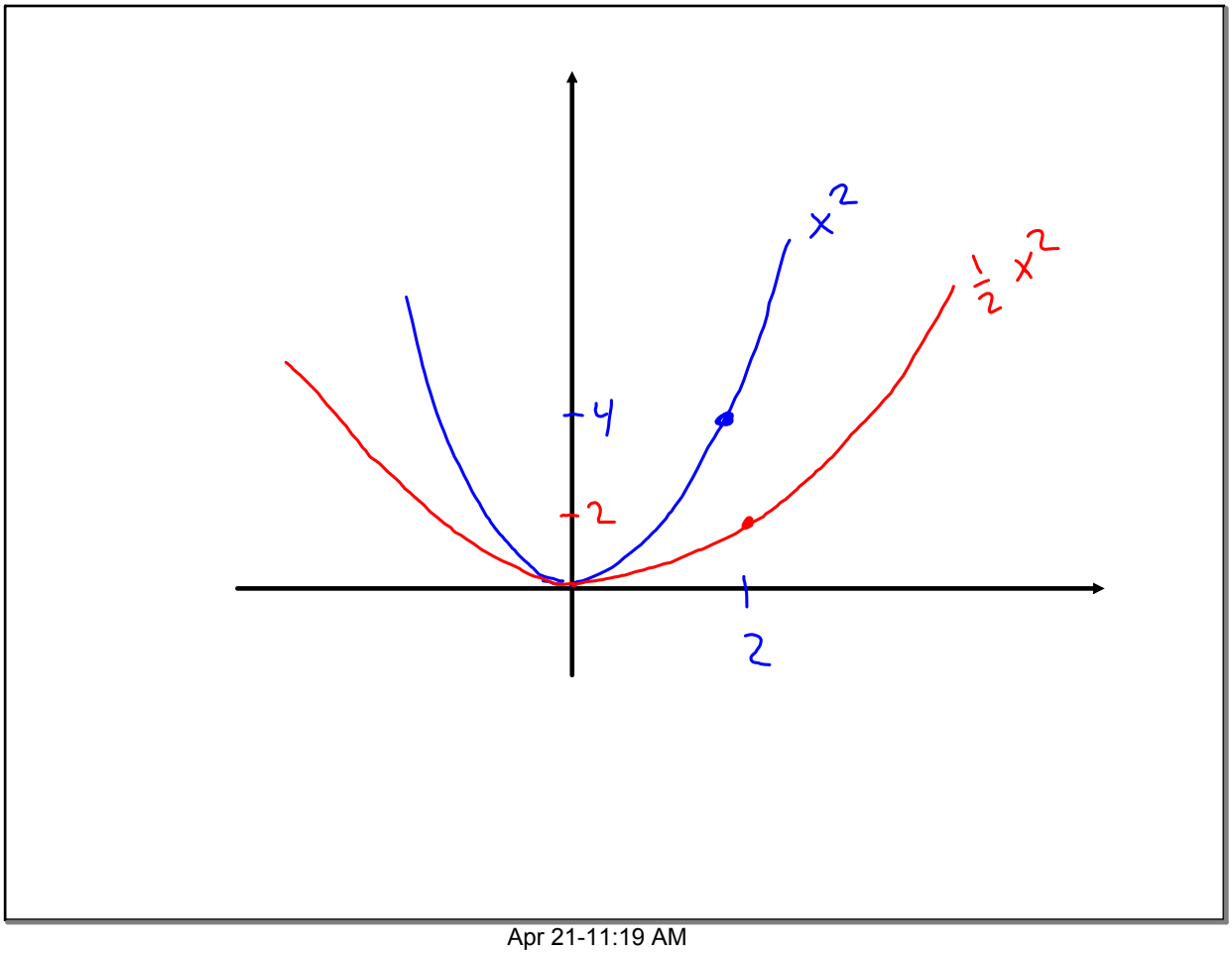

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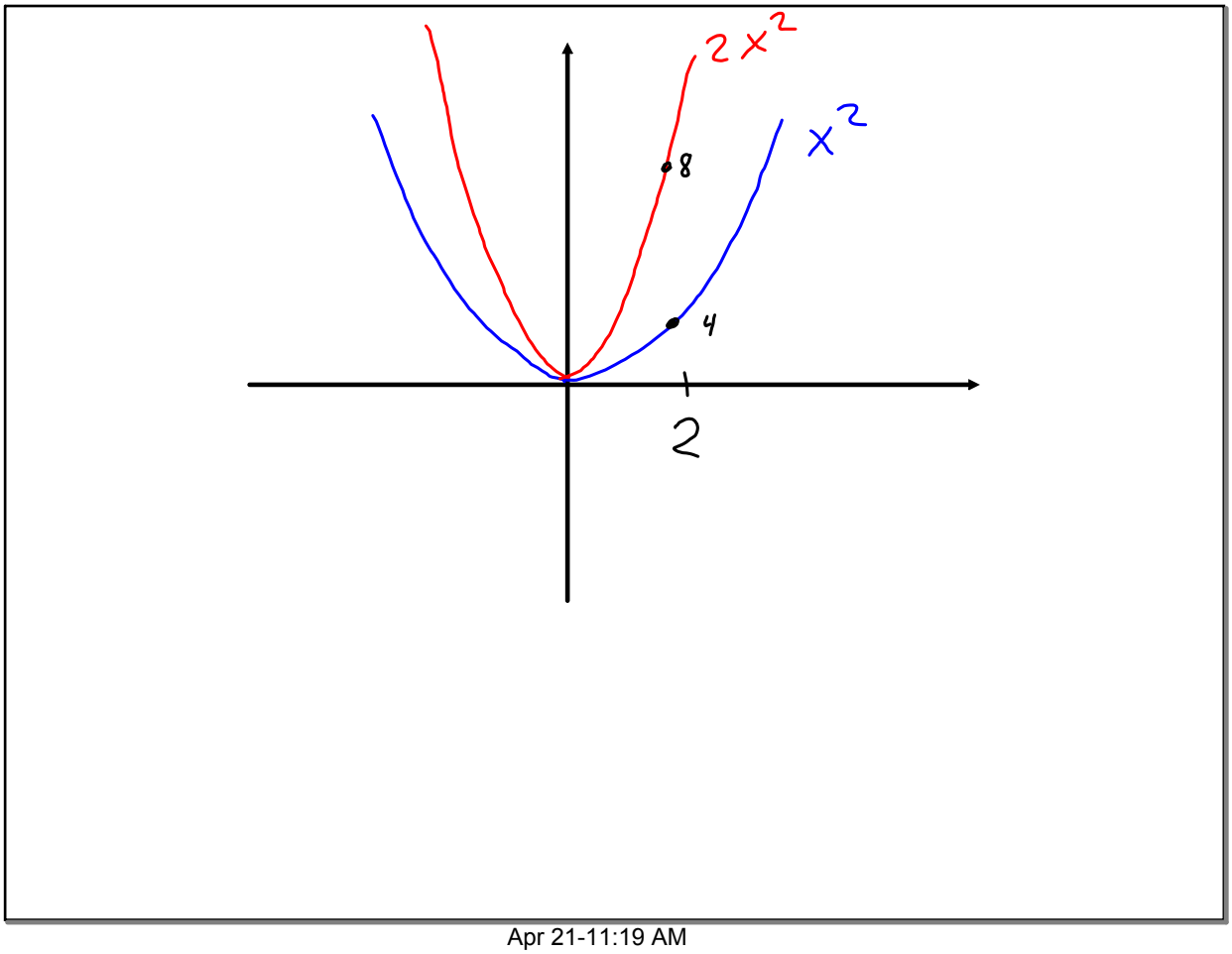

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The Standard Form of a quadratic function is:

$$f(x) = a(x - h)^2 + k$$

The vertex is at the point (h,k).

Example: $f(x) = (x - 1)^2 + 2$

a = 1
opens upward
"normal" size.

vertex is at the point (1,2)

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Key Points when graphing a parabola:

1) y-intercept
2) x-intercept(s)
3) vertex

$f(x) = (x-1)^2 + 2$

$y = f(0) = (0-1)^2 + 2$

$= (-1)^2 + 2 = 1 + 2 = 3$

y-int: $(0,3)$

Let's Graph it!

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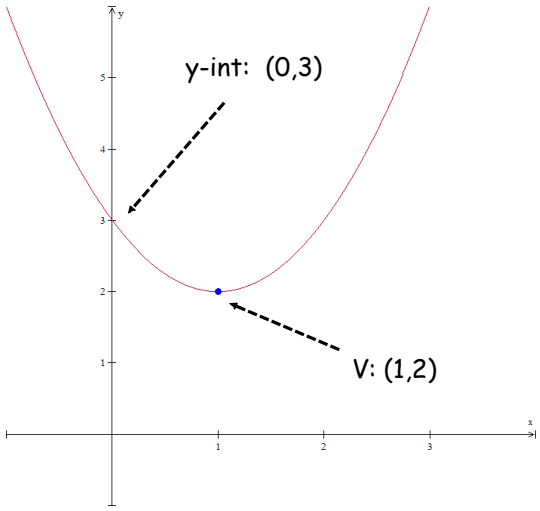

$f(x) = (x-1)^2 + 2$ Standard Form

$= (x-1)(x-1) + 2$ Convert to quadratic form using FOIL

$= x^2 - 2x + 1 + 2$

$f(x) = x^2 - 2x + 3$ Quadratic Form

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Example: Graph the quadratic function.

$f(x) = x^2 - 2x - 1$ ⟷ Quadratic Form

Convert to Standard Form by Completing the Square.

$f(x) = (x^2 - 2x \quad) - 1$

$(x^2 - 2x + 1) - 1 - 1$

$(x-1)(x-1) - 2$

$f(x) = (x-1)^2 - 2$

Standard Form

$\left(\frac{-2}{2}\right)^2 = 1$

Add/Subtract the appropriate amount

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$f(x) = (x-1)^2 - 2$ $\quad a(x-h)^2 + k$

① Identify vertex

V: $(1, -2)$

② Find the x-intercepts(the zeros)

$0 = x^2 - 2x - 1$

Convert to Quadratic Form and use the Quadratic Formula.

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)} \approx 2.414, -.414$$

or, solve by extracting square roots: $0 = (x-1)^2 - 2$

③ Find the y-intercept

$(0, f(0)) = (0, -1)$

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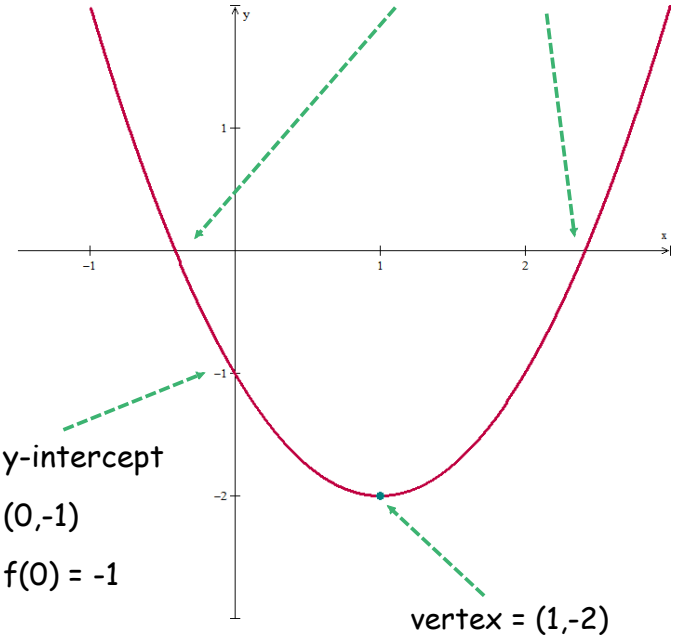
$f(x) = x^2 - 2x - 1$

④ Plot the points and sketch.

x-intercepts:

(-.414,0) (2.414,0)



y-intercept

(0,-1)

f(0) = -1

vertex = (1,-2)

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Example: Sketch the quadratic function:

$$f(x) = 2x^2 - 4x + 1$$

$(2x^2 - 4x) + 1$

Factor a out of the x^2-term and the x-term

$2(x^2 - 2x) + 1$

$2(x^2 - 2x\ \ ) + 1$

$\left(\frac{-2}{2}\right)^2 = 1$ ADD/SUBTRACT

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$2(x^2 - 2x + 1\ ) + 1 - 2$

Note: you must subtract 2 because you added 2.

$\left(\frac{-2}{2}\right)^2 = 1$

ADD/SUBTRACT the correct amount

$f(x) = 2(x^2 - 2x + 1) - 1$

$f(x) = 2(x-1)^2 - 1$

Vertex is at (1,-1)

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$f(x) = 2(x-1)^2 - 1$ $V: (1,-1)$

$a(x-h)^2 + k$

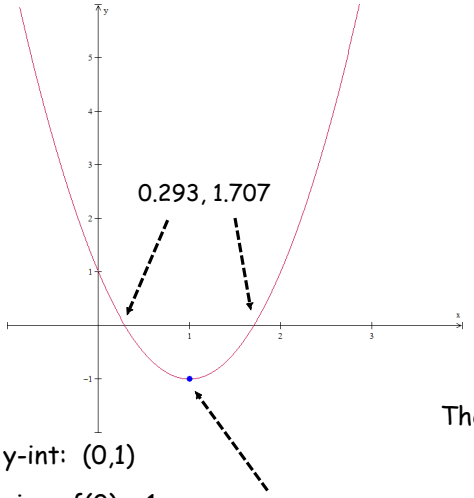
$f(x) = 2x^2 - 4x + 1$ ------ Quadratic Form.

$0 = 2x^2 - 4x + 1$ Use Q.F. to find the zeros:

$$x = \frac{-(-4) \pm \sqrt{16 - 4 \cdot 2 \cdot 1}}{2 \cdot 2} \approx 1.707, 0.293$$

or extract square roots:

$$0 = 2(x-1)^2 - 1$$
$$1 = 2(x-1)^2$$
$$\frac{1}{2} = (x-1)^2$$
$$\sqrt{\frac{1}{2}} = \sqrt{(x-1)^2}$$
$$\pm\sqrt{\frac{1}{2}} = x - 1$$
$$1 \pm \sqrt{\frac{1}{2}} = x$$



These are the zeros of f:

$0.293, 1.707 \approx x$

y-int: (0,1)

since f(0) = 1

vertex: (1,-1)

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Example: Finding a quadratic function if given:

1) The vertex: (3,4)

2) A point: (1,2)

Start with the Standard Form and fill in the vertex info:

$$f(x) = a(x-h)^2 + k$$

$$f(x) = a(x-3)^2 + 4$$ Vertex is at (h,k) = (3,4)

Use the fact: f(1) = 2 which can be written 2 =f(1)

$$2 = f(1) = a(1-3)^2 + 4$$

$$2 = a(1-3)^2 + 4$$ Solve for a.

$$2 = a(-2)^2 + 4$$
$$2 = a4 + 4$$
$$-2 = a4$$
$$-\frac{1}{2} = a$$

$$f(x) = -\frac{1}{2}(x-3)^2 + 4$$

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The End.

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