Section 2.7

Inverse Functions

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An ordered pair (x,y) ***satisfies*** a function f if

$y = f(x)$ or equivalently $f(x) = y$

example: $y = f(x) = 2x + 1$

(3,7) satisfies the function

$f(3) = 7$

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Suppose there are two functions f and g.

If $a = f(b)$ WHENEVER $b = g(a)$

then f and g are ***inverse functions***.

We also say that f and g are "***inverses of each other***".

That is:

f is the inverse of g AND g is the inverse of f

"WHENEVER" means for ANY ordered pair (b,a) that satisfies f, the ordered pair (a,b) satisfies g.

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Example: $f(x) = 3x$ $g(x) = \frac{x}{3}$

(2,6) <-- switch --> (6,2)

$f(2) = 3(2)$ $g(6) = \frac{6}{3}$

$f(2) = 6$ $g(6) = 2$

(3,9) <-- switch --> (9,3)

$f(3) = 9$ $g(9) = 3$

BUT, it must hold true for all ordered pairs, not just a few that you might pick randomly.

How can we prove this?

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...Example(cont'd): $f(x)=3x$ $g(x)=\frac{x}{3}$

Let *a* be an arbitrary number....

then

$f(a)=3a$ switch

so (a, 3a) satifies *f*. and (3a, a) satifies *g*.

check: $g(3a)=\frac{3a}{3}$

$g(3a)=a$

Since *a* was arbitrary, the property must hold for all ordered pairs.

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...Example(cont'd): $f(x)=3x$ $g(x)=\frac{x}{3}$

Notice that each function "REVERSES" the other one.

Whatever *f* "does" to *x*,

g reverses that and you get *x* back again.

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Example: Suppose $f(x) = 5x$

If *g* is the inverve of *f*, what is the formula for *g* ?

$g(x) = \frac{x}{5}$

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Example: Suppose $f(x) = x + 4$

If *g* is the inverve of *f*, what is the formula for *g* ?

$g(x) = x - 4$

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Notes:

1) Inverse functions exist in pairs: if *g* is the inverse of *f*, then *f* is the inverse of *g*.

2) A function does not have to have an inverse.

3) A function can only have ONE inverse(the inverse is unique).

4) For a function named *f*, the inverse can be written f^{-1}

Example: $f(x) = x + 4$ and $f^{-1}(x) = x - 4$

name

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Inverse functions and TABLES.

$f(x) = x + 4$ $f^{-1}(x) = x - 4$

x	-1	0	1	2	3
f(x)	3	4	5	6	7

x	3	4	5	6	7
$f^{-1}(x)$	-1	0	1	2	3

(-1,3) (0,4) (1,5) (2,6) (3,7) (3,-1) (4,0) (5,1) (6,2) (7,3)

What is an easy description of how to make a table for the inverse function? SWITCH THE ROWS

These are all ways of saying the same thing:

f has an inverse function

-OR-

f is invertible

-OR-

f^{-1} exists

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Example: Determining if the inverse exists from a table.

x	-1	-2	0	3	4
g(*x*)	5	6	7	8	9

x	5	6	7	8	9
$g^{-1}(x)$	-1	-2	0	3	4

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Example: Determining if the inverse exists from a table.

x	-1	-2	0	3	4
h(*x*)	5	6	5	8	9

x	5	6	5	8	9
$h^{-1}(x)$	-1	-2	0	3	4

So, you can say:

h does not have an inverse function

-OR-

h is not invertible

-OR-

h^{-1} does not exist

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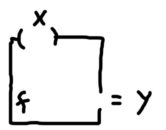
A "function box".

x goes in, something happens, y comes out.



$\Leftrightarrow$ $y = f(x)$

mathematical notation

Note: The "something" is often given by a mathematical expression.

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The Identity Function: The output equals the input.

These are all the same function:

$f(x) = x$ $\quad h(x) = x$ $\quad \left(\frac{f}{g}\right)(x) = x$

$f(t) = t$

$g(x) = x$

$k(y) = y$ $\quad g(z) = z$

$(f + g)(x) = x$ $\quad$ ...and many more.

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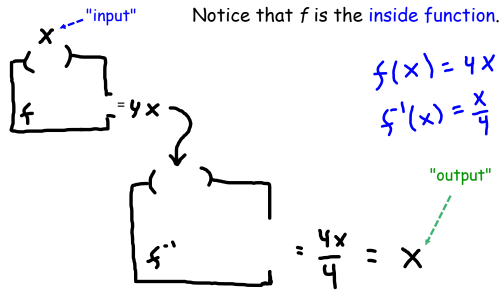
Compostion of inverse functions:

Notice that f is the inside function.

$f(x) = 4x$
$f^{-1}(x) = \frac{x}{4}$



The boxes pictured above correspond to the following:

$f^{-1}(f(x)) = x$ --OR-- $(f^{-1} \circ f)(x) = x$

"input" "output"

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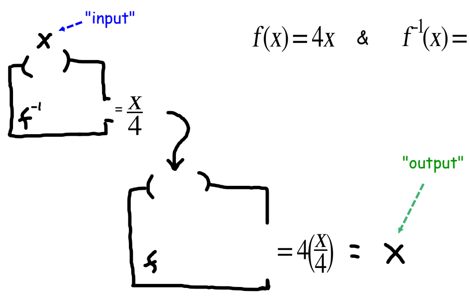
Compostion of inverse functions, reversing the order of f and f^{-1}.

$f(x) = 4x$ & $f^{-1}(x) = \frac{x}{4}$



The boxes pictured above correspond to the following:

$f(f^{-1}(x)) = x$ --OR-- $(f \circ f^{-1})(x) = x$

"input" "output"

Notice that f^{-1} is the inside function.

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Since two inverse functions "reverse" each other, composing a pair of inverse functions just gives you the "identity function".

$$(f \circ f^{-1})(x) = f(f^{-1}(x)) = x$$

-- AND --

$$(f^{-1} \circ f)(x) = f^{-1}(f(x)) = x$$

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To VERIFY that two functions *f* and *g* are inverses:

Find $f(g(x))$ and $g(f(x))$.

If $f(g(x)) = x$ --AND-- $g(f(x)) = x$

then *f* and *g* are inverse functions.

Then you may say

$g(x) = f^{-1}(x)$ -- OR -- $f(x) = g^{-1}(x)$

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Example: Verify that *f* and *g* are inverse functions.

$f(x) = 7x + 1$ $\quad g(x) = \frac{x-1}{7}$

$$f(g(x)) = f\left(\frac{x-1}{7}\right) = 7\left(\frac{x-1}{7}\right) + 1$$

$$x - 1 + 1 = x$$

$$g(f(x)) = g(7x+1) = \frac{7x+1-1}{7} = x$$

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Verify that *f* and f^{-1} are in fact inverses.

$f(x) = 4x$ & $f^{-1}(x) = \frac{x}{4}$

Note: by naming this function f^{-1} we are implying the inverse exists, even though that is what we are trying to determine.

$$(f \circ f^{-1})(x) = f(f^{-1}(x))$$

$$f\left(\frac{x}{4}\right) = 4\frac{x}{4} = x$$

$$(f^{-1} \circ f)(x) = f^{-1}(f(x))$$

$$f^{-1}(4x) = \frac{4x}{4} = x$$

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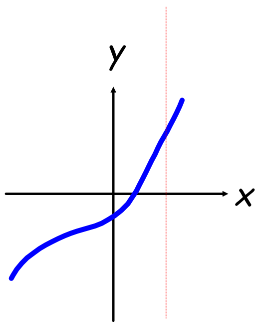
Q: How do we know if y is a function of x? (when looking at the graph)

A: The Vertical Line Test.



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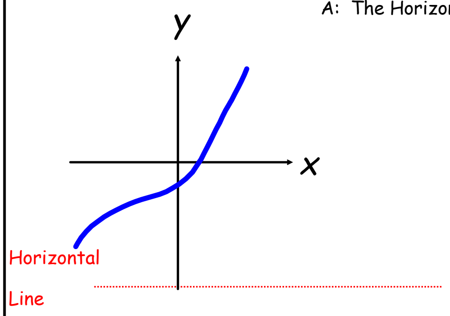
Q: How do we know if x is a function of y? (when looking at the graph)

A: The Horizontal Line Test.



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Fact: If y is a function of x

AND

x is a function of y

Then the relationship is INVERTIBLE.

In other words: There is a pair of functions which are inverses of each other.

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Testing a graph to see if it has an inverse:

1) Vertical Line Test -- tells you if y is a function of x.

2) Horizontal Line Test -- tells you if x is a function of y.

- If the graph passes both tests, it is the graph of an invertible function .

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Procedure for finding the inverse function if given an equation:

$f(x) = \frac{5x-3}{2}$

$y = \frac{5x-3}{2}$ Replace $f(x)$ with y.

$x = \frac{5y-3}{2}$ Interchange x with y.

$2x = 5y-3$ Solve for y.

$2x+3 = 5y$

$\frac{2x+3}{5} = y$

$f^{-1}(x) = \frac{2x+3}{5}$ Replace y with $f^{-1}(x)$.

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Verify they are inverses by composing the functions:

$f(x) = \frac{5x-3}{2}$ $f^{-1}(x) = \frac{2x+3}{5}$

$$f(f^{-1}(x)) = f\left(\frac{2x+3}{5}\right) = \frac{5\left(\frac{2x+3}{5}\right)-3}{2}$$

$$\frac{2x+3-3}{2} = \frac{2x}{2} = x$$

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Verify they are inverses by composing the functions in reverse order:

$f(x) = \frac{5x-3}{2}$ $f^{-1}(x) = \frac{2x+3}{5}$

Verify:

$$f^{-1}(f(x)) = f^{-1}\left(\frac{5x-3}{2}\right) = \frac{2\left(\frac{5x-3}{2}\right)+3}{5}$$

$$\frac{5x-3+3}{5}$$

$$\frac{5x}{5} = x$$

So, $f^{-1}(f(x)) = x$ is the Identity Function.

and

So, $f(f^{-1}(x)) = x$ is the Identity Function.

The two functions are inverses.

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Example: $f(x) = 2x+3$

Find the inverse function and verify that they are inverses.

$y = 2x+3$ Replace $f(x)$ with y.

$x = 2y+3$ Interchange x with y.

$x-3 = 2y$ Solve for y.

$\frac{x-3}{2} = y$

$f^{-1}(x) = \frac{x-3}{2}$ Replace y with $f^{-1}(x)$.

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...Example:

Verify that $f(x) = 2x + 3$ and $f^{-1}(x) = \frac{x - 3}{2}$ are inverses.

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One-to-One Functions

A function is called one-to-one if each output has only one input.

The only functions which have an inverse are those that are one-to-one.

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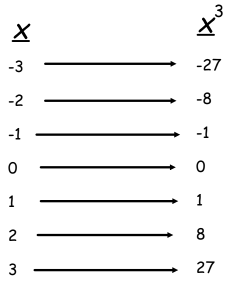
Example of a one-to-one function:

$f(x) = x^3$

x	-3	-2	-1	0	1	2	3
x^3	-27	-8	-1	0	1	8	27



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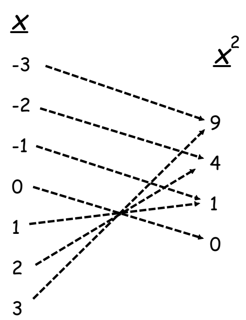
Example of a function which is NOT one-to-one:

$f(x) = x^2$

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9



You might say that this function is "two-to-one".

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The End.

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