Section 2.6

Combinations of Functions

Making new functions from old.

Nov 15-11:00 AM

A little bit of set theory....

Unions and Intersections of sets:

Suppose there are two sets(call them set A and set B).

The "Union of A with B" is a new set which consists of all the elements that appear in either A or B.

Notation: $A \cup B$ is said "A union B".

The "Intersection of A with B" is a new set which consists of all the elements that appear in both A and B.

Notation: $A \cap B$ is said "A intersect B".

Example: $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$

Note: these two sets are finite.

$A \cup B = \{1, 2, 3, 4\}$ "A union B"

$A \cap B = \{2, 3\}$ "A intersect B"

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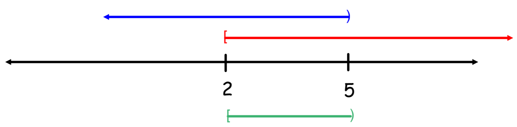
Let x be a real number....

$A = \{ 2 \leq x < \infty \}$ $B = \{ -\infty < x < 5 \}$



$A \cap B = \{ 2 \leq x < 5 \}$

$A \cap B$ = "all numbers that lie beneath both the blue and red line"

Note: these sets are infinite.

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"Arithmetic" means:

1) Adding $a + b$

2) Subtracting $a - b$

3) Multiplying ab

4) Dividing a/b

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Given any two functions f and g, the four Arithmetic Combinations are:

$(f+g)(x)=f(x)+g(x)$ Sum function: "f plus g"

$(f-g)(x)=f(x)-g(x)$ Difference function: "f minus g"

$(fg)(x)=f(x)g(x)$ Product function: "f times g"

$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$ Quotient function: "f divided by g"

Note: $(f+g)$ $(f-g)$ (fg) (f/g) are the *names* of the new functions.

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Example: $f(x)=x^2$ and $g(x)=\sqrt{4-x^2}$

The four Arithmetic Combinations are:

$(f+g)(x)=f(x)+g(x)$
$(f+g)(x)=x^2+\sqrt{4-x^2}$ Sum

$(f-g)(x)=f(x)-g(x)$
$(f-g)(x)=x^2-\sqrt{4-x^2}$ Difference

$(fg)(x)=f(x)g(x)$
$(fg)(x)=x^2\sqrt{4-x^2}$ Product

$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$ $\left(\frac{f}{g}\right)(x)=\frac{x^2}{\sqrt{4-x^2}}$ Quotient

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Let's practice using our new functions:

$(f+g)(x)=x^2+\sqrt{4-x^2}$
$(f+g)(1)=1^2+\sqrt{4-1^2}=1+\sqrt{3}$

$(f-g)(x)=x^2-\sqrt{4-x^2}$
$(f-g)(1)=1^2-\sqrt{4-1^2}=1-\sqrt{3}$

$(fg)(x)=x^2\sqrt{4-x^2}$
$(fg)(1)=x^2\sqrt{4-x^2}=1^2\sqrt{4-1^2}=1\sqrt{3}=\sqrt{3}$

$\left(\frac{f}{g}\right)(x)=\frac{x^2}{\sqrt{4-x^2}}$
$\left(\frac{f}{g}\right)(1)=\frac{1}{\sqrt{3}}$

What about $\left(\frac{f}{g}\right)(2)=$

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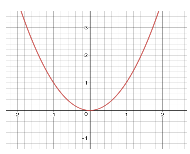
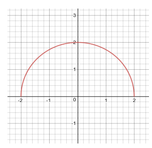
What about the domain and range?

$f(x)=x^2$

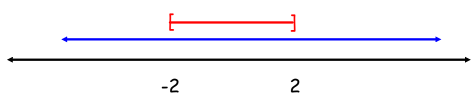


$g(x)=\sqrt{4-x^2}$



Domain of f = { $-\infty < x < \infty$ }

Domain of g = { $-2 \le x \le 2$ }



Dom f ∩ Dom g = { $-2 \le x \le 2$ }

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So, the domain of the new functions:

$$(f+g)(x)=x^2+\sqrt{4-x^2}$$ Domain of $(f+g)$ = $\{-2 \le x \le 2\}$

$$(f-g)(x)=x^2-\sqrt{4-x^2}$$ Domain of $(f-g)$ = $\{-2 \le x \le 2\}$

$$(fg)(x)=x^2\sqrt{4-x^2}$$ Domain of (fg) = $\{-2 \le x \le 2\}$

$$\left(\frac{f}{g}\right)(x)=\frac{x^2}{\sqrt{4-x^2}}$$ Domain of (f/g) = $\{-2 < x < 2\}$

We must exclude the zeros of g from the domain of the quotient function.

The "strictly inequality" does this.

Note: the zeros of g are $x = \pm 2$

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Function Composition

"Putting one function inside the other"

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$$(f \circ g)(x)=f(g(x))$$

"f composed with g of x equals f of g of x"

$(f \circ g)$ is the name of the new function

f is the *outside* function.

g is the *inside* function.

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We can reverse the order:

$$(g \circ f)(x)=g(f(x))$$

This is said "g composed with f."

It's exactly the same operation, but now f is the inside function.

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a) $f(x) = x^2$ $g(x) = 2x-1$

$(f \circ g)(x) = f(\ g(x)\ )$

$f(\ 2x-1\ ) = (2x-1)^2$

$(f \circ g)(x) = (2x-1)^2$

$(f \circ g)(3) = (2 * 3 - 1)^2 = 25$

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$(g \circ f)(x) = g(\ f(x)\ )$

$g(\ x^2\ ) = 2x^2 - 1$

$(g \circ f)(x) = 2x^2 - 1$

$(g \circ f)(3) = 2(3)^2 - 1 = 17$

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b) $f(x) = x^2 - 2x$ $g(x) = \frac{2}{3}x$

$(f \circ g)(x) = f(\ g(x)\ )$

$= f(\frac{2}{3}x\ ) = \left(\frac{2}{3}x\right)^2 - 2\left(\frac{2}{3}x\right)$

$= \frac{4}{9}x^2 - \frac{4}{3}x$

$$(f \circ g)(1) = f(g(1)) = \frac{4}{9}(1^2) - \frac{4}{3}(1) = \frac{-8}{9}$$

$(g \circ f)(x) = g(f(x))$

$g(x^2 - 2x\ ) = \frac{2}{3}(x^2 - 2x)$

$$(g \circ f)(1) = g(f(1)) = \frac{2}{3}(1^2 - 2 \cdot 1) = -\frac{2}{3}$$

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c) $h(x) = \sqrt{x-1}$ $k(x) = x^3 + 2$

$(h \circ k)(x) = h(\ k(x)\ )$

$h(x^3 + 2) = \sqrt{x^3 + 2 - 1} = \sqrt{x^3 + 1}$

$(k \circ h)(x) = k(\ h(x)\ )$

$= k(\sqrt{x-1}) = \left(\sqrt{x-1}\right)^3 + 2$

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d) $g(x) = \sqrt{x}$ $f(x) = x^2$

$(f \circ g)(x) = f(g(x))$
$= f(\sqrt{x}) = (\sqrt{x})^2 = x$

So, $(f \circ g)(x) = x$ "The Identity Function"

$(g \circ f)(x) = g(f(x))$
$= g(x^2) = \sqrt{x^2} = |x|$

This is the Absolute Value Function.

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e) $f(x) = 3x$ $g(x) = \frac{x}{3}$

$(f \circ g)(x) = f(g(x))$
$(f \circ g)(x) = f(\frac{x}{3})$
$(f \circ g)(x) = f(\frac{x}{3}) = 3(\frac{x}{3}) = x$
$(f \circ g)(x) = x$ "The Identity Function"

$(g \circ f)(x) = g(f(x))$
$(g \circ f)(x) = g(3x)$
$(g \circ f)(x) = g(3x) = \frac{3x}{3} = x$
$(g \circ f)(x) = x$ "The Identity Function"

So, $(f \circ g)(x) = x = (g \circ f)(x)$ both equal the Identity Function.

And that's special.

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Exercise 43

a) $(f+g)(x) = f(x) + g(x)$

$(f+g)(3) = f(3) + g(3)$
$= 2 + 1 = 3$

b) $(\frac{f}{g})(2) = \frac{f(2)}{g(2)} = \frac{0}{2} = 0$

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The End.

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