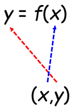
Section 2.3
Analyzing the Graphs of Functions

Oct 28-1:35 PM

The graph of a function *f* is the set of all ordered pairs (*x*,*y*) that satisfy the equation


$y = f(x)$

(x,y)

Satisfy means the function equals *y* when input is *x*.

Note: the strict definition says the graph is a set of ordered pairs, but we usually think of the graph as the picture we would see if we could graph this infinite set.

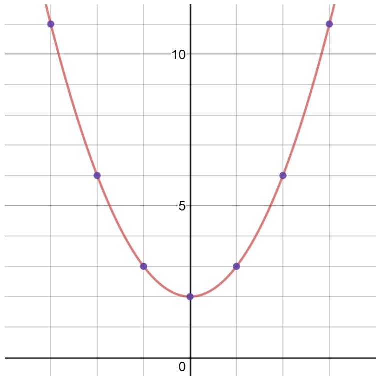
The set is made up of points (input, output) or $(x, f(x))$

Mar 19-11:17 AM

$y = f(x) = x^2 + 2$

$y = x^2 + 2$

x	-3	-2	-1	0	1	2	3
f(x)	11	6	3	2	3	6	11



The "graph" of the function *f*.

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One way to draw a graph is by *point plotting*, which means plotting several of the points that satify the function and trying to draw a line through them.

This is a poor method and is old-fashioned.

Using technology, we can easily see an accurate graph.

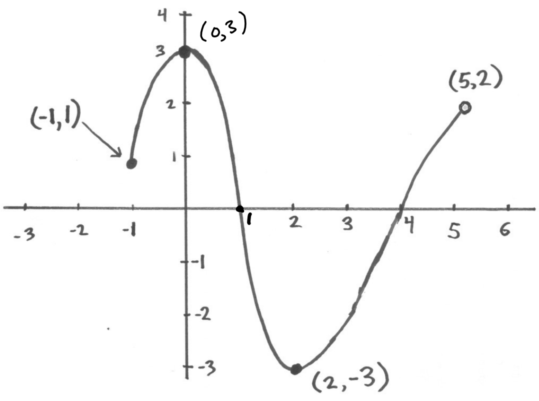
We will use Desmos.

Oct 25-3:46 PM

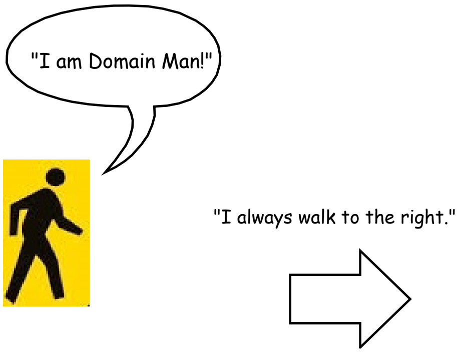
Important note: This section in the text is not about how to draw the graph of a function from its formula.

Rather, it is about identifying features of the graph and analyzing the behavior of the function based on these features.

Example from the text: How to "analyze" a graph:

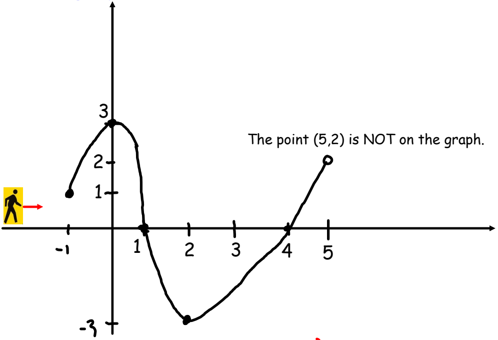


Oct 27-10:30 AM



Oct 27-2:17 PM

Ex: Finding the domain and range by looking at the graph.



The point (5,2) is NOT on the graph.

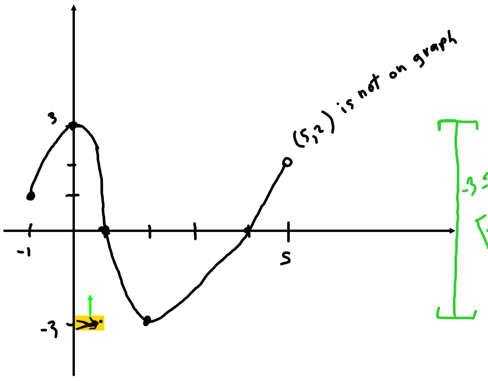
$-1 \le x < 5$

$[-1,5)$

The *domain* is the set of all *x*-values the graph takes on.

Mar 19-11:25 AM

Ex: Finding the domain and range by looking at the graph.



$-3 \le y \le 3$

$[-3,3]$

The *range* is the set of all y-values the graph takes on.

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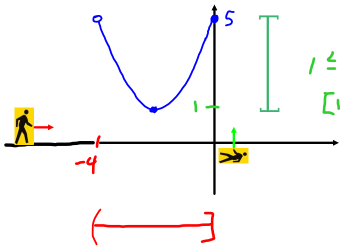
Ex: Finding the domain and range by looking at the graph.



$1 \le y \le 5$

$[1,5]$

RANGE

$-4 < x \le 0$

$(-4,0]$

DOMAIN

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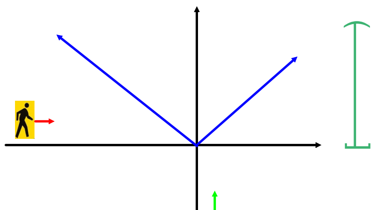
Ex: Finding the domain and range by looking at the graph.



RANGE

$0 \le y < \infty$

$[0,\infty)$

DOMAIN

$-\infty < x < \infty$

$(-\infty,\infty)$

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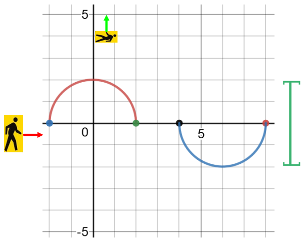
Ex: Finding the domain and range by looking at the graph.



RANGE

$-2 \le y \le 2$

$[-2,2]$

[----------] [----------]

$-2 \le x \le 2$ U $4 \le x \le 8$

DOMAIN: [-2,2] U [4,8]

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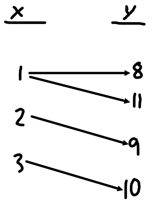
Q: Does this table describe a function from x to y?

or, "is y a function of x?"

x (input)	1	2	3	1
y (output)	8	9	10	11



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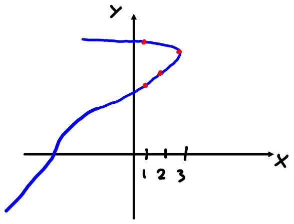
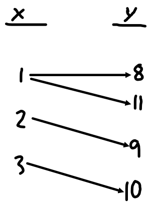
Consider a possible graph where one part *lies above* another part......

x (input)	1	2	3	1
y (output)	8	9	10	11





If any part of a graph lies above another part, that means there are two points on the graph with same x-coordinate. That means one output has two different outputs.

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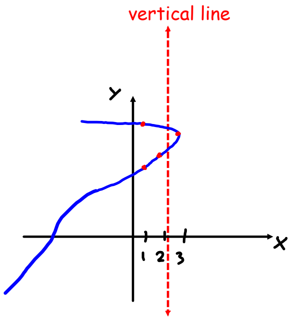
The Vertical Line Test

If a vertical line passes across a graph and touches it at multiple points at once, then y is NOT a function of x.



Note: "multiple" means "more than one"

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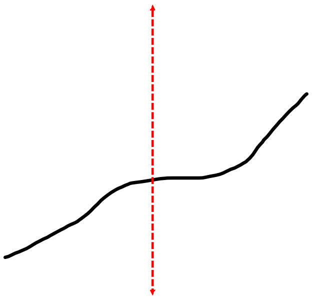
The Vertical Line Test

Or, in other words....

On the graph of any function where "y is function of x", or $y = f(x)$, it will be true that the vertical line will never intersect the graph at more than one point at a time.



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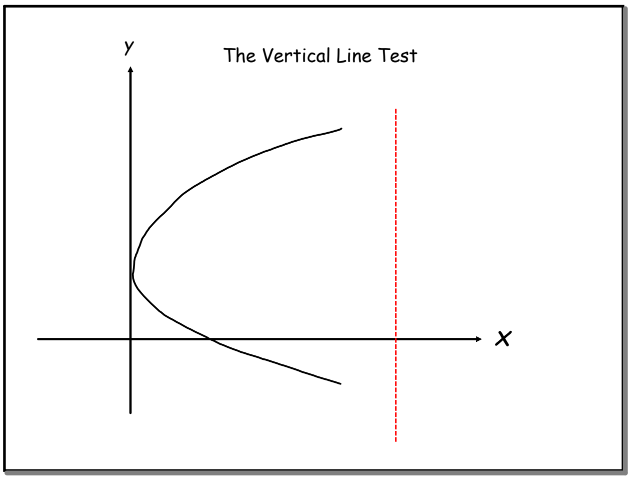
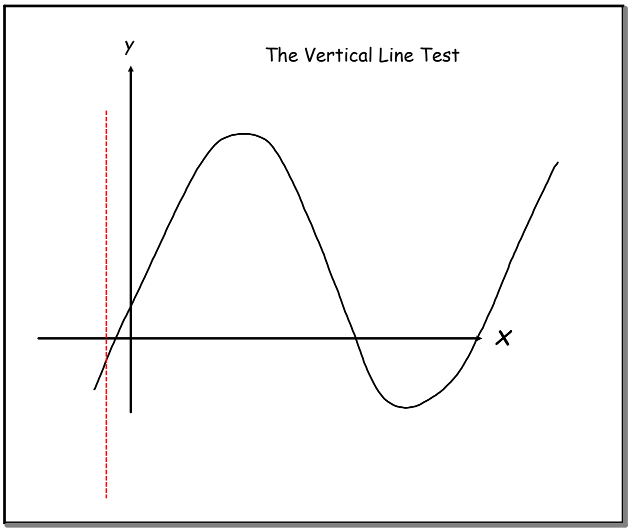
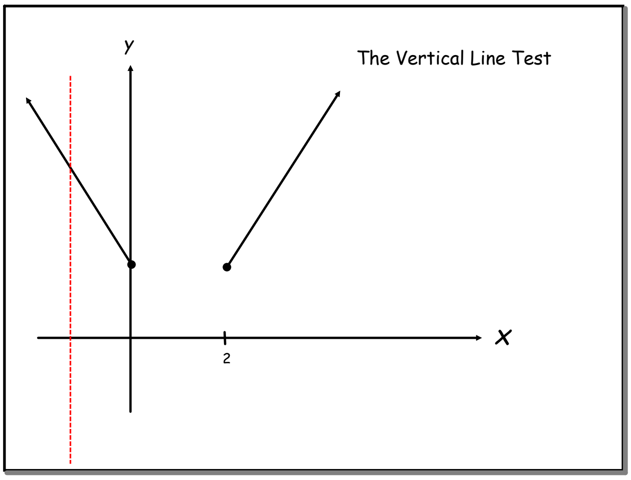

The Vertical Line Test

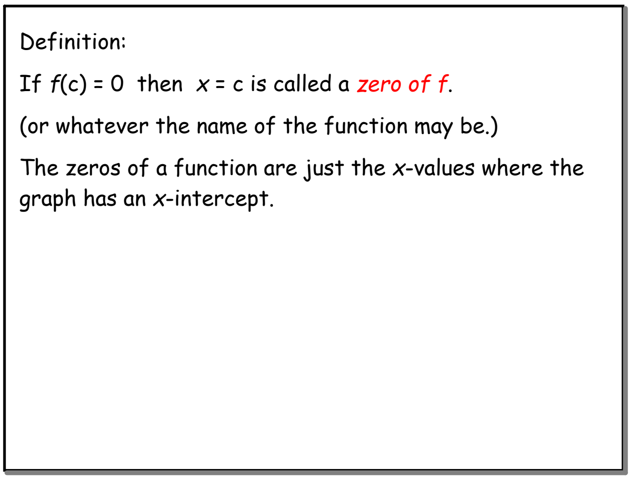
Oct 30-11:07 AM


The Vertical Line Test

Oct 30-11:07 AM


The Vertical Line Test

Oct 30-11:07 AM


Definition:

If *f*(c) = 0 then *x* = c is called a *zero of f*.

(or whatever the name of the function may be.)

The zeros of a function are just the *x*-values where the graph has an *x*-intercept.

Oct 25-3:49 PM

Finding the zeros of a function algebraically

definition

a) $g(x) = 2x+4$

To find the zeros, set the definition equal to 0 and solve.

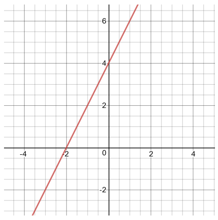
$\cancel{g(x)} = 2x+4$

0

$0 = 2x+4$

$-4 = 2x$

$-2 = \frac{-4}{2} = x$



check: $g(x) = 2x+4$

$g(-2) = 2(-2)+4$

$= 0$

$x = -2$ is the zero of g.

Mar 24-11:24 AM

Finding the zeros of a function algebraically

b) find the zeros of $f(x) = x^2-2x$

$0 = x^2-2x$

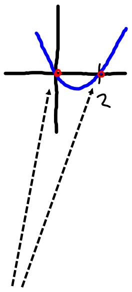
$0 = x(x-2)$

↗ ↑

$x=0$ $x=2$

check: $f(0) = 0^2-2(0) = 0$

$f(2) = 2^2-2(2) = 0$



$x = 0,2$ are the zeros of f...also known as x-intercepts.

Mar 24-11:35 AM

Finding the zeros of a function algebraically

c) $g(t) = 2t^2+4$

$0 = 2t^2+4$

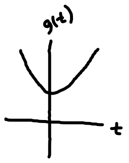
$-4 = 2t^2$

$-2 = t^2$

$\sqrt{-2} = \sqrt{t^2}$

$\sqrt{-2} = t$ "g has no zeros"

that's NOT a real number!!



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Finding the zeros of a function algebraically

d) $h(x) = \sqrt{25-x^2}$

$0 = \sqrt{25-x^2}$

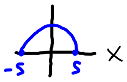
$0^2 = \left(\sqrt{25-x^2}\right)^2$

$0 = 25-x^2$

$x^2 = 25$

$\sqrt{x^2} = \sqrt{25}$

$x = \pm 5$ are the zeros of h



Mar 24-11:41 AM

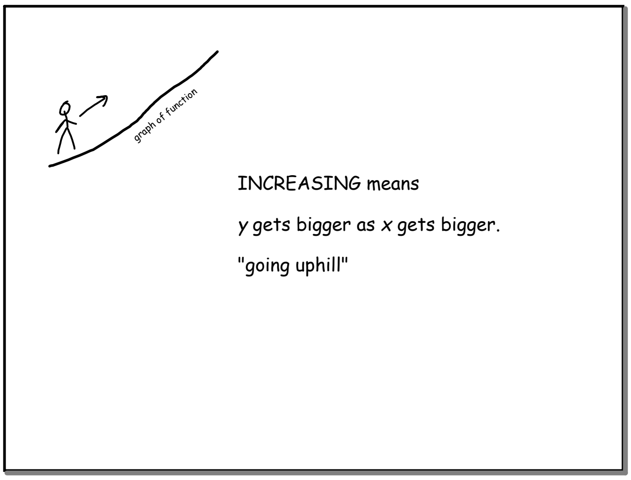
Intervals of:

Increasing

Decreasing

Constant

Mar 16-10:45 AM



INCREASING means

y gets bigger as x gets bigger.

"going uphill"

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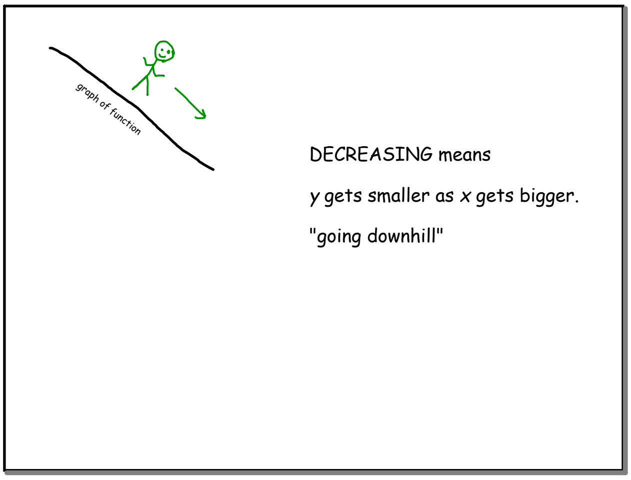

DECREASING means

y gets smaller as x gets bigger.

"going downhill"

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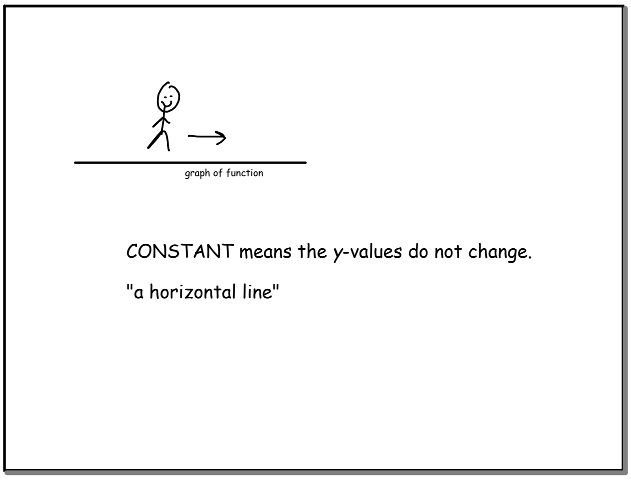


CONSTANT means the y-values do not change.

"a horizontal line"

Mar 24-11:45 AM

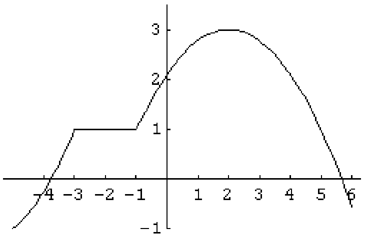
When you specify the intervals where a graph is INCREASING/DECREASING/CONSTANT you should always use the *strict inequality* symbol: $<$

Do not use the *less than or equal* to symbol: $\leq$

Apr 10-11:37 AM

Finding intervals of INC/DEC/CON

"U" means "combined with"



Increasing: (-∞,-3) U (-1,2)

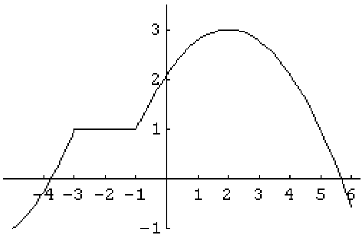
Decreasing: (2,∞)

Constant: (-3,-1)

Oct 19-1:41 PM

Finding intervals of INC/DEC/CON

"U" means "combined with"



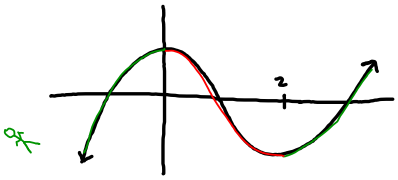
Increasing: $-\infty < x < -3$ U $-\infty < x < -3$ (-∞,-3) U (-1,2)

Decreasing: $2 < x < \infty$ (2,∞)

Constant: $-3 < x < -1$ (-3,-1)

Oct 19-1:41 PM

Finding intervals of INC/DEC/CON



INC: $-\infty < x < 0$ U $2 < x < \infty$ (-∞,0) U (2,∞) INC

DEC: $0 < x < 2$ (0,2) DEC

Constant: Nowhere

Oct 19-1:45 PM

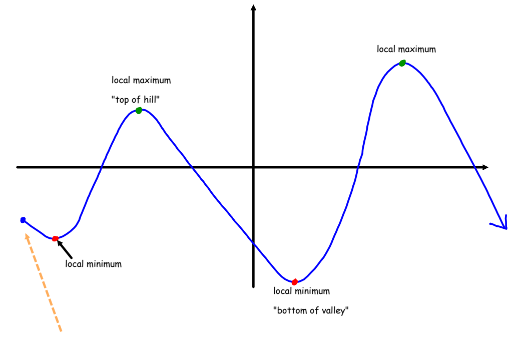
maxima and minima

Note: "*maxima*" is plural of maximum.

"*minima*" is plural of minimum.

Together, maxima or minima are called "extrema."

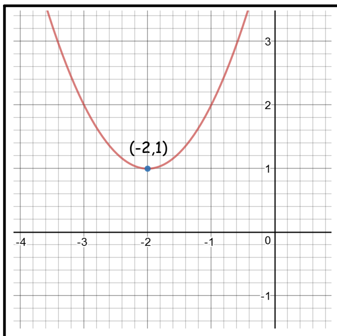
Mar 19-11:19 AM



The end point of a graph **cannot** be a local max/min

This graph has 4 local extrema.

Apr 9-10:59 AM



Identifying the location of an extremum.

This graph has a local minimum at x = -2.

Or you can say the local minimum is at (-2,1).

Apr 9-10:57 AM

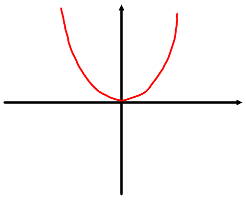
A graph can have several types of symmetry:

1) *y*-axis symmetry
2) *x*-axis symmetry
3) origin symmetry

Oct 28-1:38 PM

1) *y*-axis symmetry

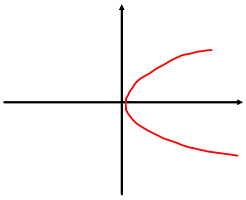
The graph is a "reflection over the *y*-axis." This means folding the graph on the *y*-axis will cause the two halves to match up.



Oct 28-1:41 PM

2) *x*-axis symmetry

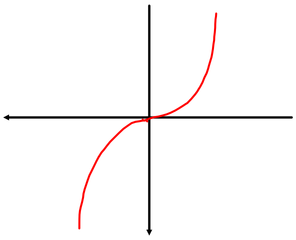
The graph is a "reflection over the *x*-axis." This means folding the graph on the *x*-axis will cause the two halves to match up.



Oct 28-1:41 PM

3) origin symmetry

Turning the graph upside down will result in the same graph.



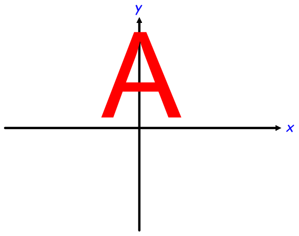
Oct 28-1:41 PM

Let's look at the letters of the alphabet and consider what kind of symmetry each exhibits.

Keep in mind the symmetry depends on where the letter is placed in the Cartesian Plane.

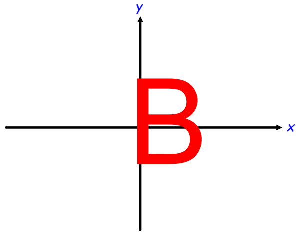
Also, the letter might not actually be the graph of a function, but it still could have symmetry.

Oct 28-1:47 PM



	y - axis symmetry
	x - axis symmetry
	origin symmetry

Mar 22-11:46 AM



	y - axis symmetry
	x - axis symmetry
	origin symmetry

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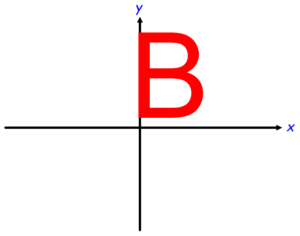

	y - axis symmetry
	x - axis symmetry
	origin symmetry

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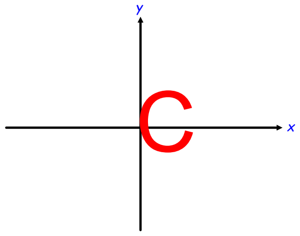

	y - axis symmetry
	x - axis symmetry
	origin symmetry

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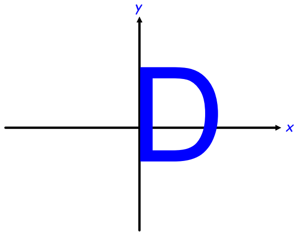

	y - axis symmetry
	x - axis symmetry
	origin symmetry

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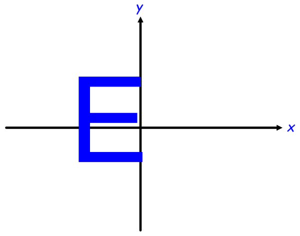

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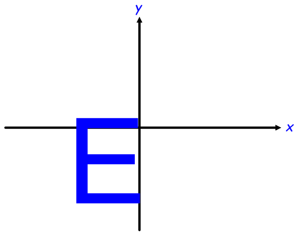

	y - axis symmetry
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	origin symmetry

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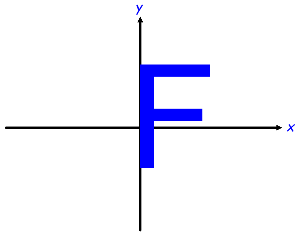

	y - axis symmetry
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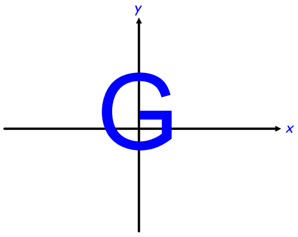
	origin symmetry

Mar 22-11:46 AM



	y - axis symmetry
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	origin symmetry

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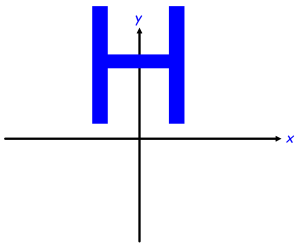

	y - axis symmetry
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	origin symmetry

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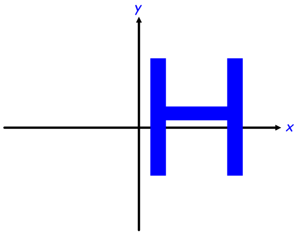

	y - axis symmetry
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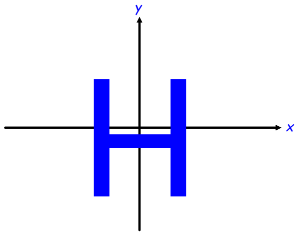

	y - axis symmetry
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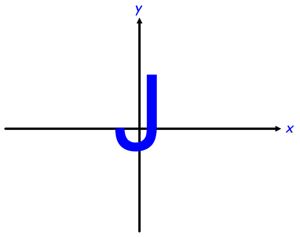

	y - axis symmetry
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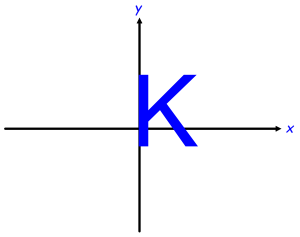

	y - axis symmetry
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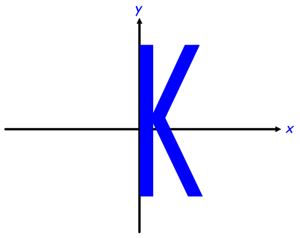
Mar 22-11:46 AM



	y - axis symmetry
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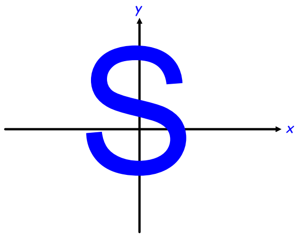
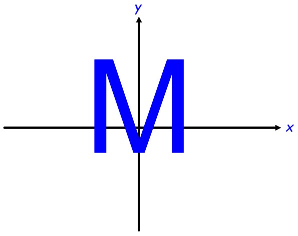
Mar 22-11:46 AM



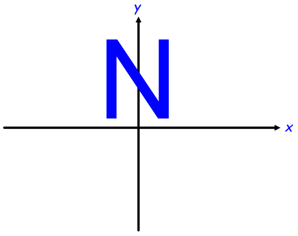
	y - axis symmetry
	x - axis symmetry
	origin symmetry

Mar 22-11:46 AM



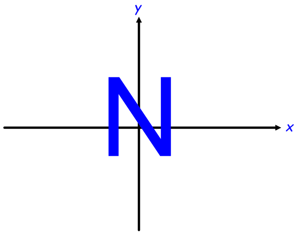
	y - axis symmetry
	x - axis symmetry
	origin symmetry

Mar 22-11:46 AM



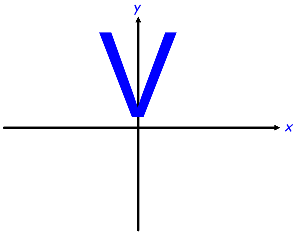
	y - axis symmetry
	x - axis symmetry
	origin symmetry

Mar 22-11:46 AM



	y - axis symmetry
	x - axis symmetry
	origin symmetry

Mar 22-11:46 AM



	y - axis symmetry
	x - axis symmetry
	origin symmetry

Mar 22-11:46 AM

EVEN and ODD functions

If $f(-x) = f(x)$ then f is an EVEN function.

"*Opposite inputs give same output.*"

If $f(-x) = -f(x)$ then f is an ODD function.

"*Opposite inputs give opposite output.*"

An alternate way of describing an ODD function:

If $f(x) = y$

then $f(-x) = -y$

Oct 28-1:50 PM

Two simple examples:

Even function $f(x) = x^2$

x	-2	-1	0	1	2
$f(x)$	4	1	0	1	4

"*Opposite inputs give same output.*"

Odd function $f(x) = x^3$

x	-2	-1	0	1	2
$f(x)$	-8	-1	0	1	8

"*Opposite inputs give opposite output.*"

Nov 5-8:33 AM

Determining if a function is *even* or *odd*

Step One: find $f(-x) =$ *expression*

Step Two: Compare the *expression* to $f(x)$ and $-f(x)$

If $f(-x) = f(x)$ then f is an EVEN function.

If $f(-x) = -f(x)$ then f is an ODD function.

If f is not even or odd, then it is NEITHER.

Oct 19-1:58 PM

Is this function *even* or *odd* ??

$$f(x) = x^4 + x^2$$

Step One: find $f(-x)$

$$f(-x) = (-x)^4 + (-x)^2$$

simplify using algebra:

$$f(-x) = (-x)(-x)(-x)(-x) + (-x)(-x)$$

$$f(-x) = (x)(x)(x)(x) + (x)(x)$$

$$f(-x) = \underbrace{x^4 + x^2}_{f(x)}$$

they are equal

Step Two:

Since $f(-x) = f(x)$

f is an EVEN function.

Oct 19-1:58 PM

Is this function even or odd ??

$f(x) = x^3 + x$

Step One: find $f(-x)$

$f(-x) = (-x)^3 + (-x)$

$f(-x) = (-x)(-x)(-x) + (-x)$

$f(-x) = -(x)(x)(x) - x$

$f(-x) = -x^3 - x$

$f(-x) = -(x^3 + x)$

$f(-x) = -(f(x)) = -f(x)$

Step Two:

Since *f*(-*x*) = -*f*(*x*)

f is an ODD function.

Oct 19-1:58 PM

Is this function even or odd ??

$f(x) = x^3 + 1$

Step One: find $f(-x)$

$f(-x) = (-x)^3 + 1$

$f(-x) = (-x)(-x)(-x) + 1$

$f(-x) = -x^3 + 1$

Step Two:

f is not an ODD function.

f is not an EVEN function.

You can say "NEITHER!"

Not EVEN

So, $f(-x) = -x^3 + 1 \neq f(x) = x^3 + 1$

and

$f(-x) = -x^3 + 1 \neq -f(x) = -x^3 - 1$

Not ODD

Oct 19-1:58 PM

Determining Symmetry from the formula:

Given a *formula* for a function, the type of symmetry can be determined using algebra.

Procedure:

Find the formula for *f*(-*x*) and see if it equals *f*(*x*) or -*f*(*x*) or neither.

Useful facts:
The graph of an even function will have *y*-axis symmetry.
The graph of an odd function will have origin symmetry.

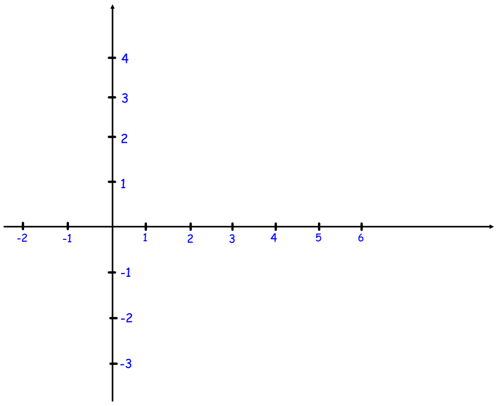
Mar 16-10:56 AM

The End.

Oct 19-2:30 PM

Junk Yard.

Nov 1-1:29 PM



Mar 19-11:25 AM