Section 2.2
Functions

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A *function from A to B* is a rule that connects each number in set A to a unique(one and only one) number in Set B.

Often times we just say "function" but we must remember that the definition involves two sets.

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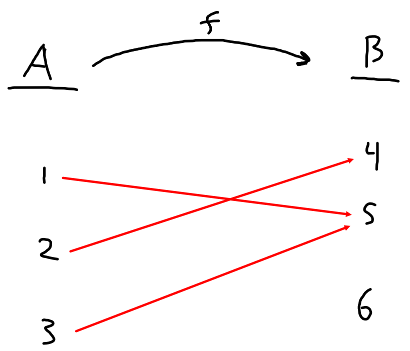
A function *f* from set A to set B.



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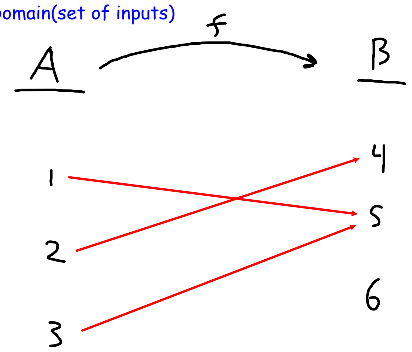
Domain(set of inputs)



Range

(set of outputs)

Domain = {1, 2, 3}

Range = {4, 5}

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The Squaring Function.

Here are some ways of describing this function:

1) WORDS: "The output is obtained by squaring the input."

This method is not very concise.

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The Squaring Function.

2) TABLE:

x(input)	-2	0	2	3
y(output)	4	0	4	9

3) A set of ordered pairs:

{ (-2,4), (0,0), (2,4), (3,9) }

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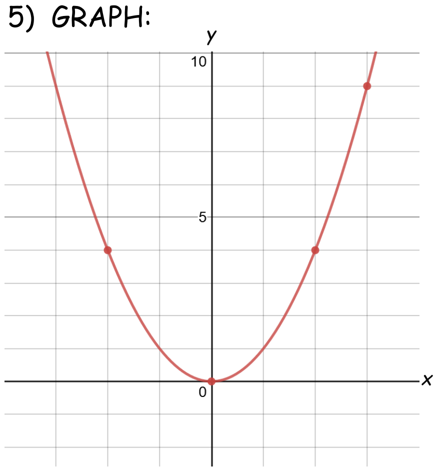
4) EQUATION: $y = x^2$

The Squaring Function.

5) GRAPH:



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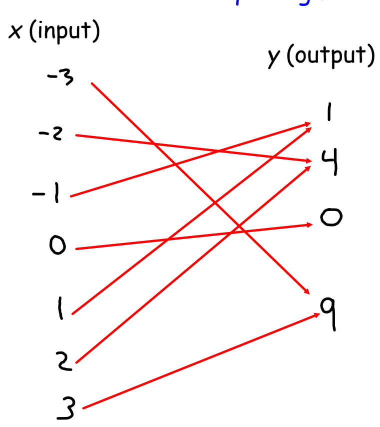
6) "PICTURES"

The Squaring Function.



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Here is another function:

The output is obtained by multiplying the input by 2 and adding 1.

"*Please represent this function in the different ways mentioned above.*"

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The same function as a table and as ordered pairs.

The output is obtained by multiplying the input by 2 and adding 1.

x	-1	0	1	2	10
y	-1	1	3	5	21

(-1,-1) (0,1) (1,3) (2,5) (10,21)

(x,y)

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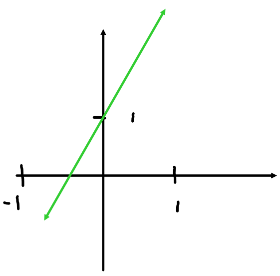
The function as an equation: $y = 2x + 1$



x	0	$-\frac{1}{2}$
y	1	0

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$y = x^2$

dependent variable

output

independent variable

input

$y = 2x + 1$

The dependent variable is the one that is ISOLATED.

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Example: Determining if "y is a function of x" from an equation.

$$x^2 + y = 1$$

We wish to determine if "y is a function of x".

1. Solve for y.

2. If the equation is solved "uniquely", then y is a function of x.

$$y = 1 - x^2$$

Yes, y is a function of x.

"Uniquely" means each value for x will result in a single value for y.

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Example: Determining if "y is a function of x" from an equation.

$$-x + y^2 = 1$$

$$y^2 = 1 + x$$

$$\sqrt{y^2} = \sqrt{1+x}$$

$$y = \pm\sqrt{1+x}$$

Q: Is y a function of x ?

1) Solve for y.

Notice the $\pm$.

Any particular value for x will result in TWO values for y.

For example:

If $x = 8$, then $y = +3$ and -3

2) NOT uniquely solved for y.

x	y
8	+3
	-3

No, y is not a function of x.

So, y is not a function of x.

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When there is a functional relationship between two variables, instead of(or in addition to) using an equation, we can use

Function Notation.

FUNCTION NOTATION

$$y = f(x)$$

Say, "y is equal to f of x"

means "y is a function of x"

FUNCTION NOTATION

$y = 2x + 1$ ← equation in two variables

$f(x) = 2x + 1$ ← function notation

" *f* " is the *name* of the function.

definition -- the expression tells you how to calculate the output.

" *f*(*x*) " is the *value* of the function, or the output.

You can use any letter to give a name to a function.

We might just as well have said: $g(x) = 2x + 1$

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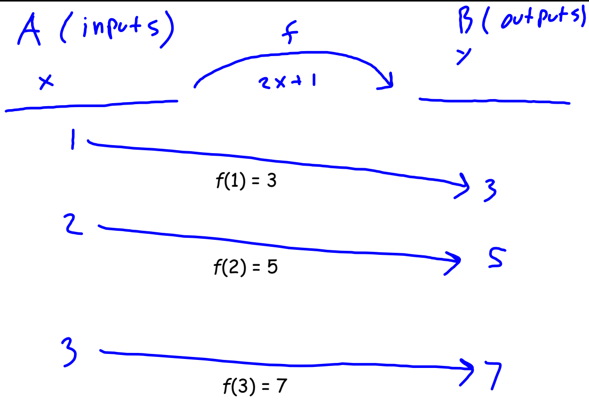

This shows how function notation corresponds to an arrow diagram.

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Practice using function notation.

$f(x) = 2x + 1$

$f(5) = 2(5) + 1$

$f(5) = 11$

$f(6) = 2(6) + 1$

$f(6) = 13$

$f(-5) = 2(-5) + 1$

$f(-5) = -9$

Whatever is between the parenthesis is the input

f(input) = 2(input)+ 1

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$f(x) = 2x + 1$

$f(a) = 2a + 1$

$f(3x) = 2(3x) + 1$

$= 6x + 1$

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DOMAIN of a Function:

The **domain** of a function is the set of all input values.

There are several ways to determine the domain of a function, depending on how the function is presented.

Recall that some ways of representing functions:

1) Tables
2) Ordered Pairs
3) Equations
4) Graphs

If you are given a **table** to represent a function then the domain is all the values in the INPUT row.

Example:

What is the domain of the function represented by this table:

x(input)	-1	0	2	5	7
y(output)	3	4	5	3	10

The domain is $x = \{ -1, 0, 2, 5, 7 \}$

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If you are given a function as set of ordered pairs then the domain is the set of all the ***first*** members of each ordered pair.

Example: What is the domain of

{ (0,1) , (2,3), (4,5), (7,6) }

The domain is { 0, 2, 4, 7 }

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Usually our functions are given as equations involving an algebraic expression.

If you are not given any other information, the domain is assumed to be the set of numbers for which the expression is defined. This is called the **implied domain**.

For our purposes, the implied domain is all x-values which DO NOT:

1) Cause division by 0.

2) Cause a negative square root.

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Example: Finding the implied domain.

$f(x) = 2x + 3$

The domain of f is:

Domain of f = $\{ -\infty < x < \infty\}$

-- or you could say --

domain of f = {all real numbers}

Another way of saying all real numbers is:

$\mathbb{R}$

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Example:

What is the domain of

$f(x) = x^2 + 2x$

dom f = { all Real numbers }

$\{ -\infty < x < \infty \} = \mathbb{R} = (-\infty, \infty)$

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Example:

What is the domain of $g(x) = \dfrac{2x}{x^2 - 4}$

Whenever $x^2 - 4 = 0$, we get division by 0, so $x = -2, 2$ are NOT in the domain of g.

Domain of g = { all x except $x = -2,2$ }

--OR--

Domain of $g = \{ x \neq -2,2 \}$

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Find the domain of $h(t) = \sqrt{2t - 1}$

The radicand must not be negative, in other words,

$2t - 1 \geq 0$

$0 \leq 2t - 1$ Solve for t.

$1 \leq 2t$

$\frac{1}{2} \leq t$

dom $h = \{ \frac{1}{2} \leq t < \infty \}$

Recall: the radicand is the expression under the radical sign.

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RESTRICTED DOMAIN:

In many applications, the domain can be restricted by physical or other practical considerations.

For example, a function giving the tax due on a purchase cannot have a negative input value.

$Tax = f(x) = 0.05x$ — x is purchase price--will never be a negative number.

Domain of $f = \{ 0 < x \}$

Note: If you were to consider the question,

What is the domain of: $f(x) = 0.05x$

you would answer Domain of $f = \{ -\infty < x < \infty \}$

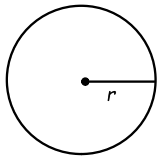
because the expression $0.05x$ is defined for all real numbers.

Domain

Example: Restricting a domain.

Let $A(r) = \pi r^2$ be the function giving the area of a circle based on its radius r.

What is the domain of the function A ?



The radius must be greater than 0, so

domain of $A = \{ 0 < r \}$

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Piecewise-Defined Functions

Sometimes a function has more than one expression that can be used to calculate the output value. The expression that you use is determined by comparing the input value to a set of inequalities.

Ex: the inequalities:

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 0 \\ x - 1 & \text{if } 0 \le x \end{cases}$$

this function has two "pieces"

$f(-1) = (-1)^2 + 1 = 2$ Use piece 1

$f(0) = 0 - 1 = -1$ Use piece 2

$f(1) = 1 - 1 = 0$ Use piece 2

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Find all real values of x such that $f(x) = 0$.

$f(x) = 5x + 1$

Set the defintion equal to 0 and solve.

$0 = 5x + 1$

$-1 = 5x$

$-\frac{1}{5} = x$

Check:

$f\left(-\frac{1}{5}\right) = 5\left(-\frac{1}{5}\right) + 1$

$f\left(-\frac{1}{5}\right) = -1 + 1$

$f\left(-\frac{1}{5}\right) = 0$

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Find all real values of x such that $f(x) = g(x)$.

$f(x) = x^2 \qquad g(x) = x + 2$

Set the definitions equal and solve.

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1, 2$

A quadratic, put in General Form and factor.

Check:

$f(-1) = 1 = g(-1)$

$f(2) = 4 = g(2)$

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The End.

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