Section 2.1

Linear Equations with Two Variables

$y = mx + b$

The variables are x and y.

m and b are constant values called 'parameters'.

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Equation with One variable

$mx + b = 0$

$x = \frac{-b}{m}$

Exactly one solution.

Equation with Two variables

$y = mx + b$

(x, y)

Number of solutions is infinite.

For example,

$y = 2x + 3$

(1,5) (2,7) (4,11)

(-3,-3) ..... are all solutions.

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Slope-Intercept Form:

$$y = mx + b$$

m is the slope

b is the y-intercept

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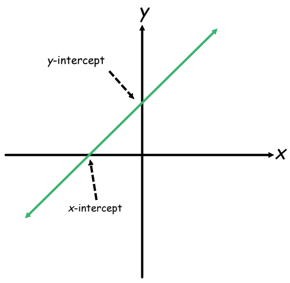
The graph is the set of points (x,y) that satisfy the equation.

The graph of a linear equation is a straight line.

x-intercept is where graph crosses x-axis. (x,0)

y-intercept is where graph crosses y-axis (0,y)



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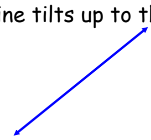
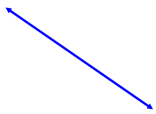
The slope of a line is a number which indicates which way the line tilts and by how much.

The slope can be positive, negative or zero.

Slope is POSITIVE ($m > 0$) Line tilts up to the right



Slope is NEGATIVE ($m < 0$) Line tilts down to the right

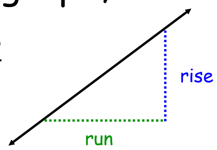


Slope is ZERO ($m = 0$) Line does not tilt

Finding the slope of a line:

1) From the graph, count

$$slope = \frac{rise}{run}$$



2) Given two points, use the formula:

point one: (x_1, y_1)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

point one: (x_2, y_2)

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A procedure to graph a line:

1) Write the equation in slope-intercept form.

2) Find the two intercepts:

x-intercept

y-intercept

3) Plot the two intercepts and draw the line between them.

x	0	
y		0

Note: If the line passes through the Origin, that is the point (0,0), then you must pick another point in the table.

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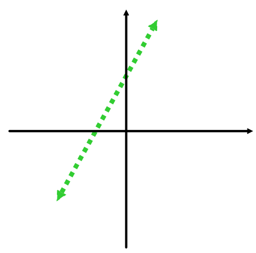
a) slope-intercept form

$y = 2x + 3$



x	0	-3/2
y	3	0

(0,3)

y-intercept

Set $y = 0$ and solve to find the *x*-intercept:

$0 = 2x + 3$

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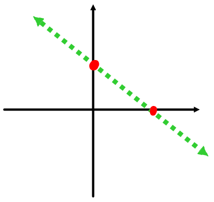
b)

$x+y=2$

$y=-x+2$

x	0	2
y	2	0



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c)

$x-y=2$

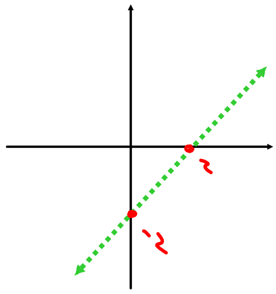
$-y=2-x$

$-(-y)=-(2-x)$

$y=-2+x$

$y=x-2$

x	0	2
y	-2	0



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d)

$2x-y=3$

$-y=-2x+3$

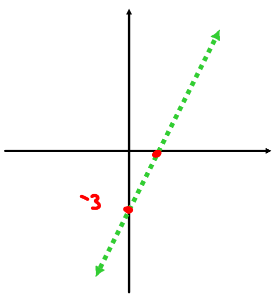
$y=2x-3$

x	0	3/2
y	-3	0

$0=2x-3$

$3=2x$

$\frac{3}{2}=x$



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e)

$y=2$

$y=0x+2$

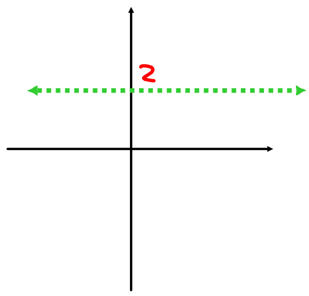
The slope is 0.

(x, y)

(anything, 2)

x	0	Und.
y	2	0

Horizontal Line



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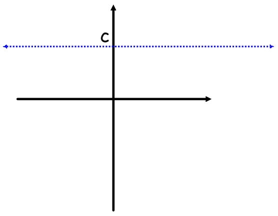
The graph of a horizontal line has the form:

$y = c$ where c is a real number.



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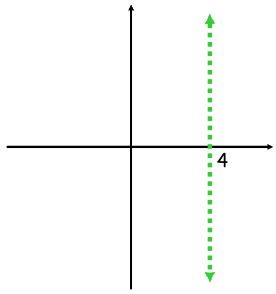
f) $x = 4$

$(4, \text{anything})$



This is a *vertical* line

and the slope is UNDEFINED.

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If c is a nonzero real number then

$\frac{c}{0}$ is UNDEFINED.

In other words, division by zero is undefined.

The special case $\frac{0}{0}$ is called INDETERMINATE.

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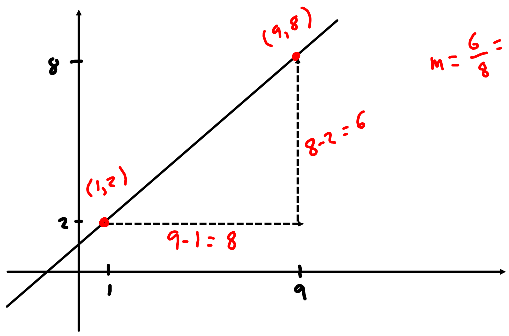

$m = \frac{6}{8} = \frac{3}{4}$

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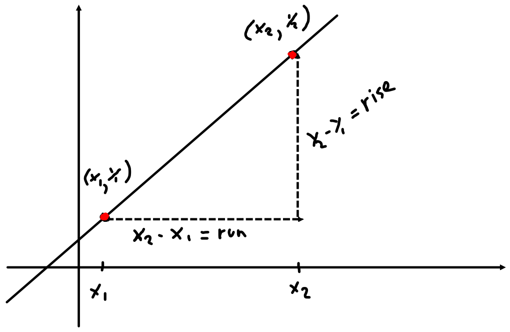

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

Different ways of referring to the slope.

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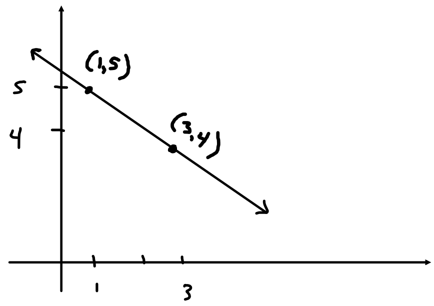
$(1,5)$ $(3,4)$ $m = \frac{4-5}{3-1} = \frac{-1}{2} = -\frac{1}{2}$



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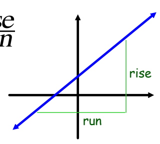
Some ways to determine the slope of line:

1) If given the slope-intercept form, identify m.

$y = mx + b$

2) If given two points, use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$

3) If given the graph, count the $\frac{rise}{run}$



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$y = mx + b$ Slope-Intercept Form

$y - y_1 = m(x - x_1)$ Point-Slope Form

(x_1, y_1) is a point on the line.

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Finding equation of a line if you know ONE POINT and the SLOPE.

Example: Find the slope-intercept EQUATION of the line passing through the point (1,2) with slope $\frac{3}{4}$.

(x_1, y_1)

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 2 = \frac{3}{4}(x - 1)$ Fill in slope and point

$y = \frac{3}{4}x - \frac{3}{4} + 2$

$y = \frac{3}{4}x + 1\frac{1}{4}$ slope-intercept form

--or--

$y = 0.75x + 1.25$ ...with the numbers as decimals.

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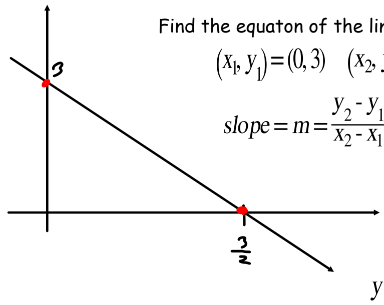
Find the equaton of the line graphed below:



$(x_1, y_1) = (0, 3)$ $(x_2, y_2) = \left(\frac{3}{2}, 0\right)$

$slope = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{\frac{3}{2} - 0} = -\frac{3}{\frac{3}{2}} = -2$

$y - y_1 = m(x - x_1)$

$y - 3 = -2(x - 0)$

$y = -2x + 3$

RULE: $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$

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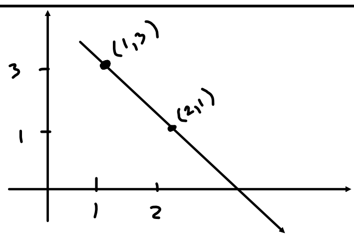

$m = \frac{1-3}{2-1} = -2$

$y = m(x - x_1) + y_1$

$y = -2(x - 2) + 1$

$y = -2x + 4 + 1$

$y = -2x + 5$

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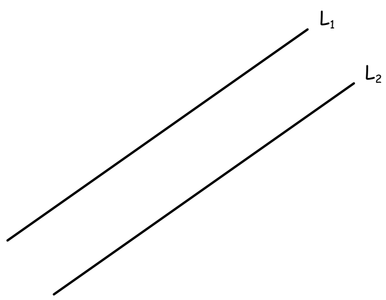
Parallel lines have the same slope:

If L_1 is parallel to L_2 then $m_1 = m_2$

m_1 is slope of L_1



m_2 is slope of L_2

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Example: Find the equation of Line 2 if it is parallel to Line 1 and Line 2 passes through (1,4).

L_1 : $y = 2x + 3$ Slope of L_1

$m_1 = 2$ Slope of L_2

L_2 is parallel to L_1 . Therefore, $m_2 = 2$

$y = m_2(x - x_1) + y_1$ (x_1, y_1)

$(1, 4)$

$y = 2(x - 1) + 4$

$y = 2x - 2 + 4$

$y = 2x + 2$ Equation of L_2

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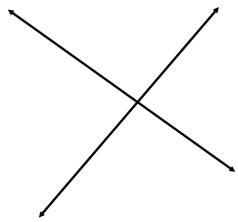
<u>Perpendicular lines:</u>

The slopes are negative reciprocals.

If the slope of L_1 is m_1

then the slope of L_2 is $m_2 = -\frac{1}{m_1}$



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Let's practice with "negative reciprocals"

m	negative reciprocal -(1/m)
2	$-\frac{1}{2}$
-3	$\frac{1}{3}$
$\frac{1}{2}$	-2
-0.5	2
2.5	$-\frac{1}{2.5}$
0	

Example: Find the equation L_2 if:

L_2 is perpendicular to L_1 and passes thru (1,4).

L_1 has equation $y = 2x + 3$

The slope of L_2 is $m_2 = -\frac{1}{2}$

$y = m_2(x - x_1) + y_1$

$y = -\frac{1}{2}(x - 1) + 4$

$y = -\frac{1}{2}x + \frac{1}{2} + 4$

$y = -\frac{1}{2}x + 4\frac{1}{2}$

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The

General Form

for the

equation of a line:

$$Ax + By + C = 0$$

Note: The General Form is NOT unique.

Any given line has an infinite number of representations in General Form.

Example: Equation of a line in the General Form

$Ax + By + C = 0$

$-6x + 3y - 9 = 0$

$3y = 6x + 9$

$\frac{3y}{3} = \frac{6x + 9}{3}$

$y = 2x + 3$

Let's write the equation in Slope-Intercept Form

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The End.

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