Section 1.6

"Other" types of equations

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So far, we have learned to solve:

1) Linear equations

2) Quadatic equations

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Linear equations have exactly one solution.

$$ax + b = 0$$

$$ax = -b$$

$$\frac{ax}{a} = -\frac{b}{a}$$

$$x = \frac{-b}{a}$$

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4 methods of solving Quadratic equations: $ax^2 + bx + c = 0$

1) Factor

2) Taking square roots

3) Complete the Square

4) Quadratic Formula

The solutions could be REAL or IMAGINARY.

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EX: Solving a polynomial equation. (To find all solutions, be sure to **factor**.)

$$3x^4 = 48x^2$$

$$3x^4 - 48x^2 = 0$$ Put all terms on one side.

$$3x^2(x^2 - 16) = 0$$ Factor.

$$3x \cdot x(x+4)(x-4) = 0$$ Factor some more.

$x = \{ 0, 0, -4, 4 \}$

The solutions are the x-values that make each factor equal to zero.

$x = 0$ is called a ***repeated solution***, because it appears twice in the factorization.

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FACT:
The number of complex solutions(real or imaginary) to a polynomial equation equals the degree.

As such, the number of real solutions of a polynomial equation is less than or equal to the degree.

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Ex: How you can lose solutions by dividing.

$$\frac{3x^4}{3x^2} = \frac{48x^2}{3x^2}$$

$$x^2 = 16$$

$$\sqrt{x^2} = \sqrt{16}$$

$$x = \pm 4$$

So, avoid dividing by an expression containing the variable.

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A useful rule about exponents:

$$(x^n)^m = x^{nm}$$

Ex: $(x^2)^3 = x^{2 \cdot 3} = x^6$

$$\left(x^{\frac{a}{b}}\right)^{\frac{c}{d}} = x^{\frac{ac}{bd}}$$ Same rule but expressed using rational exponents.

Ex: $\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = x^{\frac{6}{6}} = x^1 = x$

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Raising both sides to an exponent

$$LHS = RHS$$

$$(LHS)^{\square} = (RHS)^{\square}$$

LHS = "left hand side"

RHS = "right hand side"

You must use the same exponent on both sides.

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Ex: Solving by raising to a rational exponent:

rational means a fraction.

$$4x^{3/2} - 8 = 0$$

isolate the exponential $x^{3/2}$

$$4x^{3/2} = 8$$

$$\frac{4x^{3/2}}{4} = \frac{8}{4}$$

$$x^{3/2} = 2$$

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... Solving by raising to a rational exponent:

$$\left(x^{\frac{3}{2}}\right)^{\frac{2}{3}} = (2)^{\frac{2}{3}}$$

$$x^{\frac{3\cdot 2}{2\cdot 3}} = 2^{2/3}$$

$$x^{6/6} = 2^{\frac{2}{3}}$$

$$x^{1} = 2^{\frac{2}{3}}$$

$$x = 2^{\frac{2}{3}}$$

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base$^{\text{exponent}}$

How to evaluate this expression on your calculator: use the exponent key.

$\boxed{\text{base}}$ $\boxed{y^x}$ $\boxed{\text{exponent}}$ $=$

"exponent key"

$$2\ \boxed{y^x}\ (2 \div 3) \approx 1.587$$

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A note about radicals:

$$\sqrt{x} = \sqrt[2]{x} = x^{\frac{1}{2}}$$

$$\sqrt[3]{x} = x^{\frac{1}{3}}$$

$$\sqrt[n]{x} = x^{1/n}$$

n is called the *index* and if it does not appear it is 2 by default.

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Examples:

$$\sqrt{9} = 9^{\frac{1}{2}} = 3$$

$$\sqrt[3]{8} = 8^{1/3} = 2$$

$$\sqrt[3]{-27} = (-27)^{1/3} = -3$$

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Solve using rational exponents:

$$x^3 + 512 = 0$$

$$x^3 = -512$$

$$(x^3)^{1/3} = (-512)^{1/3}$$

$$x = x^{\cancel{3}} = (-512)^{1/3} = \sqrt[3]{-512} = -8$$

But remember the "fact" about polynomials. There should be 3 solutions in total.

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What about the two missing solutions? Please see "Special Factors" inside the book cover. In particular:

$$x^3 + a^3 = (x+a)(x^2 - ax + a^2)$$ "Sum of Two Cubes"

$$x^3 + 8^3 = (x+8)(x^2 - 8x + 64)$$

$$(x+8)(x^2 - 8x + 64) = 0$$

$$x + 8 = 0 \qquad x^2 - 8x + 64 = 0$$

$$x = -8 \qquad x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(64)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{-192}}{2} \qquad x = 4 \pm 4\sqrt{3}i$$

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Ex: Solving by squaring both sides

$$\sqrt{x-10} - 4 = 0$$

$$\sqrt{x-10} = 4$$

$$\left(\sqrt{x-10}\right)^2 = (4)^2$$

$$x-10 = 16$$

$$x = 26$$

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check

$$\sqrt{26-10} - 4 \stackrel{?}{=} 0$$

$$\sqrt{16} - 4 \stackrel{?}{=} 0$$

$$4 - 4 \stackrel{?}{=} 0$$

$$0 = 0$$

Yes, $x = 26$ is the solution!

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An "*extraneous solution*" is a solution that does not satisfy the original equation. Squaring both sides, raising both sides to a rational exponent and multiplying by a variable can introduce extraneous solutions. If you do these operations, check the potential solutions in the original equation.

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Example of an extraneous solution created by squaring both sides.

$x = 2$ one solution

$(x)^2 = (2)^2$ Square both sides

$x^2 = 4$ two solutions

$x = 2, -2$

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Example of an extraneous solution:

$\sqrt{x-5} + 2 = 0$

$\sqrt{x-5} = -2$

$(\sqrt{x-5})^2 = (-2)^2$

$x-5 = 4$

$x = 9$

Check $x = 9$ in the original equation.

$\sqrt{9-5}+2=0$

$\sqrt{4}+2=0$

$2+2=0$

$4\neq0$ --------→ $x = 9$ is NOT a solution.

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Ex: Solving an absolute value equation:

$|x-3| = 2$

Equation 1:

$x-3 = 2$

$x = 5$

Equation 2:

$x-3 = -2$

$x = 1$

The *potential* solutions are $x = 5, 1$

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$\boxed{\text{Check Solutions}}$

$|x-3| = 2$

$x = 5$

$|5-3| \stackrel{?}{=} 2$

$|2| = 2$

$2 = 2$

ok

$x = 1$

$|1-3| \stackrel{?}{=} 2$

$|-2| = 2$

$2 = 2$

ok

The solutions are $x = 5, 1$

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$2|x-3| = 8$ Isolate the | | part first.

$\frac{\cancel{2}|x-3|}{\cancel{2}} = \frac{8}{2}$

$|x-3| = 4$ Then create the two equations.

Equation 1:

$x-3 = 4$

$x = 7$

Equation 2:

$x-3 = -4$

$x = -1$

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Compounding: this is when you are paid interest on the principal you have in an account.

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$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

Type of Compounding	*n* =
Yearly	1
Semi-annually	2
Quarterly	4
Monthly	12
Daily	365

n is the number of compoundings per year.

P is the principal

r is the APR(in decimal)

t is the number of years

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$$A = P\left(1+\frac{r}{n}\right)^{n\cdot t}$$

$P = 750$

$r = .05$

$n = 4$

$t = 10$

$$750\left(1+\frac{.05}{4}\right)^{4\cdot 10}$$

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c) $A = 750\left(1+\frac{r}{4}\right)^{4\cdot 10}$

Q: what value for r will cause $A = 1500$?

Solve $1500 = 750\left(1+\frac{r}{4}\right)^{40}$

First, isolate the exponential part $\left(1+\frac{r}{4}\right)^{40}$

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$$\frac{1500}{750} = \frac{750\left(1+\frac{r}{4}\right)^{40}}{750}$$

$$2 = \left(1+\frac{r}{4}\right)^{40}$$

$$(2)^{\frac{1}{40}} = \left[\left(1+\frac{r}{4}\right)^{40}\right]^{1/40}$$

$$2^{1/40} = \left(1+\frac{r}{4}\right)^{40\cdot\frac{1}{40}}$$

$$2^{1/40} = \left(1+\frac{r}{4}\right)^{1}$$

$$2^{1/40} = 1+\frac{r}{4}$$

$$2^{\frac{1}{40}} - 1 = \frac{r}{4}$$

$$4\left(2^{\frac{1}{40}}-1\right) = \left(\frac{r}{4}\right)4$$

$$4\left(2^{\frac{1}{40}}-1\right) = r$$

4(2 ^ (1÷40) − 1) = .0699

$r = .0699 \Rightarrow$ APR 6.99%

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The End.

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