Section 1.5

Complex Numbers

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$x^2 = -1$

Can this equation have a solution if

x is a *real number*?

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No! And here is why:

$x^2 = -1$

$x \cdot x = -1$

(neg)(neg) = positive

(pos)(pos) = positive

(0)(0) = 0

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To find solutions to such an equation, we need the *imaginary unit* called *i*.

i has the property that

$i^2 = -1$

or

$i = \sqrt{-1}$

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A "Complex Number in Standard Form" has the form

$$a + bi$$

real part imaginary part

Notes:
1) a and b are both real numbers.
2) If $a = 0$, there is no real part and the number is called "*purely imaginary*."
3) If the $b = 0$, there is no imaginary part and the number is simply a real number.
4) Any number with an imaginary part is considered an imaginary number.

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Classify the complex number as real, imaginary, purely imaginary.

a) 7

$7 + 0\,i$

$a + b\,i$

Real number

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Classify the complex number as real, imaginary, purely imaginary.

b) $2 + 3\,i$

$a + b\,i$

Imaginary number

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Classify the complex number as real, imaginary, purely imaginary.

c)

$-2i$

$0 + (-2)\,i$

$a + b\,i$ Purely Imaginary number

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Classify the complex number as real, imaginary, purely imaginary.

d)

$4 + i$

$4 + (1)\, i$

$a \;+\; b\, i$

Imaginary number

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Classify the complex number as real, imaginary, purely imaginary.

e)

$-3 - 5i$

$-3 + (-5)\, i$

$a \;+\; b\, i$

Imaginary number

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Classify the complex number as real, imaginary, purely imaginary.

f)

πi

$0 + (\pi)\, i$

$a \;+\; b\, i$ Purely Imaginary number

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Classify the complex number as real, imaginary, purely imaginary.

g)

$$\frac{i}{3}$$

$0 + \frac{1}{3}i$

$a \;+\; b\, i$ Purely Imaginary number

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Classify the complex number as real, imaginary, purely imaginary.

h) $2 - 9i$

imaginary

$2 + (-9)i$

$a = 2$ $b = -9$

$a + bi$ Imaginary number

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Operations on complex numbers proceed as if i were a variable (just remember that $i^2 = -1$).

To add or subtract complex numbers:

-- add or subtract the real parts

-- add or subtract the imaginary parts

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a) $(5-i) + (2+4i)$

$5 - i + 2 + 4i$

$5 + 2 - i + 4i$

$7 + 3i$

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b) $3i + (2+4i)$

$2 + 7i$

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c) $5-(2+4i)$

$5-2-4i$

$3-4i$

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d) $(5-4i)+(2+4i)$

$5+2\ \underbrace{-4i+4i}_{0}$

7

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Multiplying complex numbers:

a) $-3(2+4i)$

$-6-12i$

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b) $i(2+4i)$

$2i+4i^2$

$2i+4(-1)$

$-4+2i$ Standard Form

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c) $(5-i)(2+9i)$

$10 + 45i - 2i - 9(i^2)$

$10 + 43i - 9(-1)$

$10 + 43i + 9$

$19 + 43i$

Use FOIL if both numbers are binomials.

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d) $(2+9i)(2-9i)$

$4 - 18i + 18i - 81i^2$

$4 \quad 0 \quad -81(-1)$

$4 \quad + 81$

85

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Complex Conjugates are two complex numbers whose imaginary parts differ only by signs(±).

$(a+bi)(a-bi)$

$a^2 - abi + abi - b^2i^2$

$a^2 \quad -b^2(-1)$

$a^2 + b^2$

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$(a+bi)(a-bi) = \underbrace{a^2 + b^2}_{\text{a positive Real number}}$

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Example of:

Multiplying by a "special form of 1"

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \underbrace{\frac{\sqrt{2}}{\sqrt{2}}}_{1} = \frac{1 \cdot \sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}}{2}$$

In this case, the objective is to rationalize the denominator.

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To simplify a fraction involving imaginary denominator, multiply by

$$\frac{a+bi}{c+di} \cdot \overbrace{\frac{c-di}{c-di}}^{1} \qquad \frac{\text{conjugate of denominator}}{\text{conjugate of denominator}}$$

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Perform the operation and write the answer in standard form. $\frac{5+3i}{2-9i}$

$$\frac{5+3i}{2-9i} \cdot \frac{2+9i}{2+9i}$$

$$\frac{(5+3i)(2+9i)}{(2-9i)(2+9i)} = \frac{10+45i+6i+27i^2}{85}$$

$$= \frac{10+51i-27}{85} = \frac{-17+51i}{85}$$

$$\frac{-17}{85} + \frac{51}{85}i$$

$a + bi$

standard form

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Perform the operation and write the answer in standard form. $\frac{2+3i}{4+i}$

$$\frac{2+3i}{4+i} \cdot \frac{4-i}{4-i} = \frac{(2+3i)(4-i)}{17}$$

$$\frac{8-2i+12i-3i^2}{17}$$

$$\frac{11+10i}{17} = \frac{11}{17} + \frac{10}{17}i$$

$a + bi$

standard form

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Perform the operation and write the answer in standard form. $\frac{2+3i}{5i}$

$$\frac{2+3i}{5i} \quad \frac{-\cancel{5}i}{-\cancel{5}i}$$

$$\frac{2+3i}{5i} \cdot \frac{i}{i} = \frac{2i+3i^2}{5i^2} = \frac{2i+3(-1)}{5(-1)}$$

$$\frac{2i-3}{-5} = \frac{-3+2i}{-5}$$

$$\frac{-3}{-5} + \frac{2i}{-5}$$

$$\frac{3}{5} - \frac{2}{5}i$$

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Algebra rule: $\sqrt{xy} = \sqrt{x}\sqrt{y}$

$$\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$$

Hint: You can verify the result with your calculator.

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Let $a > 0$. "let a be greater than zero."

$$\sqrt{-a} = \sqrt{(-1)a}$$

$$= \sqrt{-1}\sqrt{a}$$

$$i\sqrt{a}$$

The Principal Square Root of -a

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Examples of writing a negative square root as an imaginary number:

$$\sqrt{-3} = i\sqrt{3}$$

$$\sqrt{-5} = i\sqrt{5}$$

$$\sqrt{-9} = i\sqrt{9} = i\,3 = 3i$$

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Solve using the Quadratic Formula:

$3x^2 - 2x + 5 = 0$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(5)}}{2(3)} = \frac{2 \pm \sqrt{-56}}{6}$

$= \frac{2 \pm i\sqrt{56}}{6} = \frac{2 \pm i\sqrt{4 \cdot 14}}{6} = \frac{2 \pm 2i\sqrt{14}}{6}$

$= \frac{2}{6} \pm \frac{2i\sqrt{14}}{6}$

$\frac{1 \pm \sqrt{14}\, i}{3}$

$= \frac{1}{3} \pm \frac{\sqrt{14}}{3} i$

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The End.

Feb 6-8:02 PM

Attachments

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