Section 1.4

Quadratic Equations

Feb 7-11:01 AM

What is a *quadratic equation*?

The short answer: A polynomial where the biggest exponent is two.

Definition: Quadratic equation in

General Form(Quadratic Form)

$$ax^2 + bx + c = 0$$

a, *b* and *c* are called *coefficients*

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Write the quadratic equation in General(Quadratic) Form.

Identify the coefficients *a*, *b*, and *c*.

a) $2x^2 + 3x - 7 = 0$

$a = 2$

$b = 3$

$c = -7$

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Write the quadratic equation in General(Quadratic) Form.

Identify the coefficients *a*, *b*, and *c*.

b) $2x^2 = 5x - 70$

$2x^2 - 5x + 70 = 0$

$a = 2$

$b = -5$

$c = 70$

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Write the quadratic equation in General(Quadratic) Form.

Identify the coefficients *a*, *b*, and *c*.

c) $x^2 = 9$

$x^2 - 9 = 0$

a = 1

b = 0

c = -9

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Write the quadratic equation in General(Quadratic) Form.

Identify the coefficients *a*, *b*, and *c*.

d) $2x^3 + 3x - 7 = 0$

This is NOT a quadratic equation. Notice the exponent of 3.

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Write the quadratic equation in General(Quadratic) Form.

Identify the coefficients *a*, *b*, and *c*.

e) $2x^2 = 5x - 70 + 2(3 + x^2)$

Combine like terms.

$2x^2 = 5x - 70 + 6 + 2x^2$

$-2x^2 \qquad -2x^2$

$0 = 5x - 64$

This is actually a linear equation.

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Write the quadratic equation in General(Quadratic) Form.

Identify the coefficients *a*, *b*, and *c*.

f) $x^2 = 9x$

$x^2 - 9x = 0$

a = 1

b = -9

c = 0

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Write the quadratic equation in General(Quadratic) Form.

Identify the coefficients *a*, *b*, and *c*.

f) $\frac{1}{x^2} = 9x$

$x^{-2} = 9x$

$x^{-2} - 9x = 0$

Not quadratic because the exponent is in the denominator(or negative).

Recall -- a rule about exponents:

$x^{-n} = \frac{1}{x^n}$ and likewise $x^n = \frac{1}{x^{-n}}$

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The Zero-Factor Property:

If a * b = 0

then a = 0 -OR- b = 0

(It's possible both equal 0.)

The idea is: If you can write the quadratic equation as the product of two linear factors then you can solve by setting each linear factor equal to 0.

The solutions are the values that make the factors equal 0.

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Example: Solve by factoring.

$2x^2 + 9x + 7 = 3$

$2x^2 + 9x + 4 = 0$

Before factoring, put all terms on one side so it equals 0.

$(2x + 1)(x + 4) = 0$

Factor into linear factors.

$2x + 1 = 0$ $x + 4 = 0$

Set each factor equal to 0 and solve.

$x = -\frac{1}{2}$ $x = -4$

Remember: The solutions are the values that make the factors equal 0.

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Example: Solve by factoring.

$x^2 - 10x = -9$

$x^2 - 10x + 9 = 0$

$(x - 9)(x - 1) = 0$

The solutions are *x* = 1, 9

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Example: Solve by factoring.

$$2x^2 + x - 15 = 0$$

$$(2x - 5)(x + 3) = 0$$

set each linear factor = 0

and solve:

$$2x - 5 = 0 \quad x + 3 = 0$$

The solutions are: $x = \frac{5}{2}, x = -3$

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First we will solve by factoring:

$$x^2 - 4 = 0$$

$$(x + 2)(x - 2) = 0$$

$$x = -2 \quad x = 2$$

The solutions are $x = -2, 2$ or $x = \pm 2$

Note: this type of equation is called the "difference of two squares" and is an example of a "Special Factor". Please refer to inside cover of text.

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Now we will solve by ***extracting square roots***.

This means taking the square root of each side.

$x^2 = 4$ add 4 to both sides

$\sqrt{x^2} = \sqrt{4}$ Take square root of both sides

$|x| = 2$

$x = \pm 2$

Note: $\sqrt{x^2} = |x| = \pm x$

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In general:

Note: $d \geq 0$

$x^2 = d$

$\sqrt{x^2} = \sqrt{d}$

$x = \pm\sqrt{d}$

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Solve by extracting square roots:

$$x^2 = 27$$

$$\sqrt{x^2} = \sqrt{27}$$

Use the rule: $\sqrt{ab} = \sqrt{a}\sqrt{b}$

$$x = \pm\sqrt{27}$$

$$= \pm\sqrt{3 \cdot 9} = \pm\sqrt{3}\sqrt{9}$$

$$= \pm 3\sqrt{3} \approx \pm 5.196$$

"exact" solutions

numerical solutions

"exact" solutions are also called "algebraic" solutions.

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Solve by extracting square roots:

$$(x-2)^2 = 11$$ Isolate the $(...)^2$

$$\sqrt{(x-2)^2} = \sqrt{11}$$ "Square Root" both sides.

$$x-2 = \pm\sqrt{11}$$

$$x = 2 \pm \sqrt{11} \approx 5.317, -1.317$$

exact solution — numerical solutions

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Exercise:

Write $(x-2)^2 = 11$ in General Form $ax^2 + bx + c = 0$

$$(x-2)^2 = 11$$

$$(x-2)(x-2) = 11$$

Use FOIL

$$x^2 - 4x + 4 = 11$$

$$x^2 - 4x - 7 = 0$$ Minus 11 from each side.

Q: Can this be solved by factoring?

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$$x^2 - 4x - 7 = 0$$

A: No, cannot be solved by factoring.

But we did solve it by taking square roots.

We will use the method of

Completing the Square

to convert the General Form to a form that can be solved by taking square roots.

$x^2 - 4x - 7 = 0$ --- Complete the Square ---> $(x-2)^2 = 11$

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How to complete the square:

Please see handout.

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By completing the square for the equation

$ax^2 + bx + c = 0$ we obtain the

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula provides the solutions to a quadratic equation.

Jan 20-11:41 AM

Use the Quadratic Formula to solve:

$$3x^2 + 5x - 7 = 0$$

Guidelines:

1) Make sure the equation is in General Form.
2) Identify the coefficients *a*, *b*, and *c*.
3) Substitute them into the formula and simplify.

$$x = \frac{-(5) \pm \sqrt{5^2 - 4 \cdot 3 \cdot (-7)}}{2 \cdot 3} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{109}}{6}$$ Algebraic solutions

$$x \approx 0.907,\ -2.574$$ Numerical solutions

The End

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