Section 1.2

Linear Equations in One Variable

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$x^2 = 6 + x$

$x=-1$: $(-1)^2 \stackrel{?}{=} 6 + (-1)$

$1 \neq 5$

No, -1 is not a solution.

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$x = 3$:

$3^2 \stackrel{?}{=} 6 + 3$

$9 = 9$

Yes, 3 is a solution.

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$x = -3$:

$(-3)^2 \stackrel{?}{=} 6 + (-3)$

$9 \neq 3$

NO

No, -3 is not a solution.

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$x = 5$:

$5^2 \stackrel{?}{=} 6 + 5$

$25 \neq 11$

NO

No, 5 is not a solution.

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$x = -2$:

$(-2)^2 \stackrel{?}{=} 6 + (-2)$

$4 = 4$

Yes, -2 is a solution.

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The solution set is:

$x = \{3, -2\}$

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The symbol $\sqrt{}$

is called a *radical sign.*

The expression under it is called the *radicand.*

Example: $\sqrt{x - 2}$

The radicand is $x - 2$

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The *domain of an expression* is the set of all valid values allowed for the variable involved.

There are two ways a value can FAIL to be in the domain:

1) If that value causes a denominator to be zero.

2) If it causes a negative value under a square root sign(or any even rooted radical).

Otherwise, the value is VALID, and it IS IN THE DOMAIN.

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a) $\sqrt{x-5}$

Note: the radicand is the expression $x - 5$

"The radicand must not be negative."

$0 \le x - 5$

$5 \le x$

domain = $\{ 5 \le x < \infty \}$

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b) $\frac{5}{x-2}$

"You must not divide by 0."

$x-2 \neq 0$

$x \neq 2$

domain = $\{$ all x except $x=2 \}$

$= \{ x \neq 2 \}$

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a) $2x = x + x$

$2x = 2x$

Identically equal

Yes, the equation is an identity.

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b) $2x-4 = 2(x-2)$

$2x-4 = 2x-4$

Identically Equal

Yes, the equation is an identity.

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c) $x^2 = 6 + x$

If you can find even one value for *x* which does not satisfy the equation, then the equation in NOT an identity.

Let $x = 0$:

$0^2 \neq 6 + 0$

$0 \neq 6$

No, the equation is not an identity.

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The number of solutions to an equation.

An identity has an infinite number of solutions.

A conditional equation has a finite number of solutions.

A contradiction is an equation that has NO solutions.

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Definition: Linear Equation

A *linear equation* in one variable *x* is one that can be written as

$$ax + b = 0$$

where $a \neq 0$.

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Examples of linear equations in one variable:

a) $2x + 3 = 0$

$a = 2 \quad b = 3$

b) $5x - 3 = 0$

$a = 5 \quad b = -3$

c) $-3x = 0$

$a = -3 \quad b = 0$

d) $2x + 3 = -7x + 2$

$9x + 1 = 0$

$a = 9 \quad b = 1$

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Q: How many solutions are there to a linear equation?

$$ax + b = 0$$
$$\underline{\quad -b \quad -b}$$
$$ax = -b$$
$$\frac{ax}{a} = \frac{-b}{a}$$
$$x = \frac{-b}{a}$$

Exactly one solution.

This is the unique solution to a linear equation in one variable.

Note: "unique" means one and only one.

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Two equations are *equivalent* if they have exactly the some solution set.

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Generating equivalent equations:

1) Remove symbols of grouping, combine like terms, etc.

2) Add or Subtract same amount to / from BOTH SIDES.

3) multiply or DIVIDE Both sides by the same amount. (as long as that amount is not zero)

4) Switch the two sides.

e.g. $x = 2$ is the same as $2 = x$

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Solve for x:

a) $4x - 8 = 0$

$+8 \quad +8$

$4x = 8$

$\frac{4x}{4} = \frac{8}{4}$

$x = 2$

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b) $2x + 2 = -3$

$-2 \quad -2$

$\frac{2x}{2} = \frac{-5}{2}$

$x = \frac{-5}{2}$

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c) $16x - 4 = 8x + 2$

$-8x \quad -8x$

$8x - 4 = 2$

$+4 \quad +4$

$\frac{8x}{8} = \frac{6}{8}$

$x = \frac{3}{4}$

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d) $3x = 3x + 1$

$-3x \quad -3x$

$0 = 1$

but $0 \neq 1$

No Solution!

This equation is an example of a CONTRADICTION.

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e) $6(x-1)+4 = 3(7x+1)$

$6x-6+4 = 21x+3$

$6x-2 = 21x+3$

$-6x \quad -3 \qquad -6x \quad -3$

$\frac{-5}{15} = \frac{15x}{15}$

$-\frac{1}{3} = x$

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f) $x+8 = 2(x+4)-x$

$x+8 = 2x+8-x$

$x+8 = x+8$

IDENTICALLY EQUAL

This equation is an identity!

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The Least Common Denominator(LCD)

of a set of fractions is the smallest expression that each denominator divides evenly into.

To eliminate fractions, we can multiply both sides of an equation by the LCD.

Multiplying by the LCD:

The LCD = 6.

a) $\frac{x}{2} + \frac{x}{3} = 4$

$6\left(\frac{x}{2} + \frac{x}{3}\right) = 4 * 6$

$3x + 2x = 24$

$5x = 24$

$x = \frac{24}{5}$

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Multiplying by the LCD:

The LCD = 12.

b) $\frac{x}{3} + \frac{2x}{4} = 4$

$12\left(\frac{x}{3} + \frac{2x}{4}\right) = 12(4)$

$\frac{12x}{3} + \frac{12 \cdot 2x}{4} = 48$

$4x + 6x = 48$

$10x = 48$

$x = \frac{48}{10} = 4.8$

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Cross Multiplying

$\frac{a}{b} = \frac{c}{d} \Longleftrightarrow ad = cb$

"is equivalent to"

as long as $b \neq 0$ & $d \neq 0$

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a) $\frac{x}{4} = \frac{5}{2}$

$2x = 4 \cdot 5$

$2x = 20$

$x = 10$

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b) $\frac{2}{x-3} = \frac{3}{x+1}$

$2(x+1) = 3(x-3)$

$2x+2 = 3x-9$

$-2x \quad -2x$

$2 = x - 9$

$+9 \quad +9$

$11 = x$

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An *extraneous* solution is one that does not satisfy the original equation.

Some ways to create an extraneous solution:

1) Squaring both sides

2) Multiplying or dividing by a factor containing a variable expression.

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Notice:

$(x-2)(x+2)=x^2-4$

Least Common Denominator

$$\frac{1}{x-2}=\frac{3}{x+2}-\frac{6x}{x^2-4}$$

$$\frac{(x-2)(x+2)}{1}\cdot\frac{1}{x-2}=\left(\frac{3}{x+2}-\frac{6x}{x^2-4}\right)\cdot\left(\frac{x^2-4}{1}\right)$$

$$x+2=3(x-2)-6x$$

$$x+2=3x-6-6x$$

$$x+2=-3x-6$$

$$4x=-8$$

$$x=-2$$

Not in the domain of the original equation.

x = -2 is an extraneous solution.

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The End.

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