Chapter 4 Rational Functions and Conics

Section 4.2 Graphs of Rational Functions

Section Objectives: Students will know how to sketch the graph of a rational function.

I. Analyzing Graphs of Rational Functions (pp. 341-343)   Pace: 15 minutes

- Draw attention to the **Guidelines for Analyzing Graphs of Rational Functions** and the *Technology* feature on page 341 of the text.

Example 1. Sketch the graph of each of the following functions.
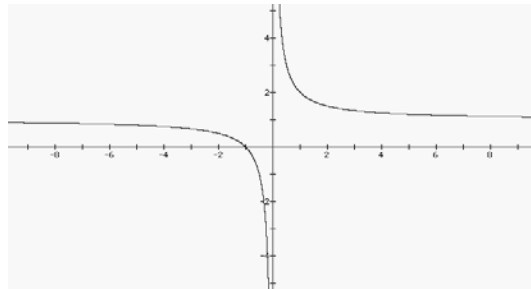
a) $f(x) = \dfrac{x+1}{x}$.

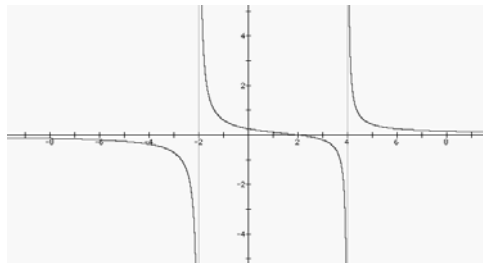|	
y-Intercept:	None
x-Intercept:	(-1, 0)
Vertical asymptote:	$x = 0$
Horizontal asymptote:	$y = 1$
Additional points:	(-2, 0.5), (-1.5, 1/3), (1, 2)

b) $g(x) = \dfrac{x-2}{x^2 - 2x - 8}$

y-Intercept:	(0, 0.25)
x-Intercept:	(2, 0)
Vertical asymptotes:	$x = -2$ and $x = 4$
Horizontal asymptote:	$y = 0$
Additional points:	(-4, -0.375), (6, 1/4)

c) $h(x) = \dfrac{x}{x^2 + 1}$
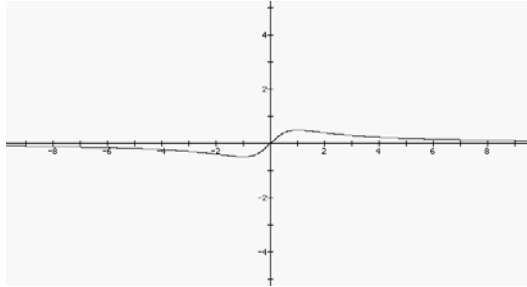
y-Intercept:	(0, 0)
x-Intercept:	(0, 0)
Vertical asymptote:	none
Horizontal asymptote:	$y = 0$
Additional points:	(-2, -0.4), (-1, -1/2), (1, 1/2)

II. Slant Asymptotes (p. 344)   Pace: 10 minutes

- Add one more rule to the **Rules for Asymptotes of a Rational Function** from Section 4.1.

  If $n = m + 1$, then the graph of f has a slant asymptote at $y = q(x)$, where $q(x)$ is the quotient obtained from the division algorithm.

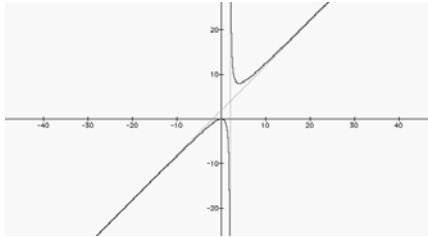**Example 2.** Sketch the graph of $y = \dfrac{x^2}{x - 2} = x + 2 + \dfrac{4}{x - 2}$

y-Intercept:	(0, 0)
x-Intercept:	(0, 0)
Vertical asymptote:	$x = 2$
Slant asymptote:	$y = x + 2$
Additional points:	(-1/2, -0.1), (1, -1), (3, 9)

III. Application (p. 345)   Pace: 5 minutes

Example 3. The cost of producing x units is $C = 0.25x^2 + 5x + 78$. The average cost per unit is
$$\overline{C} = \dfrac{0.25x^2 + 5x + 78}{x} = 0.25x + 5 + \dfrac{78}{x}.$$
Find the number of units that should be produced to minimize the average cost. Graph this function on a graphing utility, then use the "minimum" command. $x \approx 17.66$

- Assign the *Writing About Mathematics* on page 345 of the text.