Chapter 4 Rational Functions and Conics

Course/Section
Lesson Number
Date

Section 4.1 Rational Functions and Asymptotes

Section Objectives: Students will know how to determine the domains and find the asymptotes of rational functions

I. Introduction (p. 332) Pace: 5 minutes

- State the following definition.
  A **rational function** is a function of the form $f(x) = N(x)/D(x)$, where N and D are both polynomials. The domain of f is all x such that $D(x) \neq 0$.

Example 1. Find the domain of $f(x) = \dfrac{2x+1}{x^2-4}$.

$$x^2 - 4 = 0$$
$$(x+2)(x-2) = 0$$
$$x + 2 = 0 \Rightarrow x = -2$$
$$x - 2 = 0 \Rightarrow x = 2$$

The domain is $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

II. Horizontal and Vertical Asymptotes (pp. 333-334) Pace: 15 minutes

- Use the graph of $y = \dfrac{2x-5}{x-2}$ to discuss vertical and horizontal asymptotes.
- State the following definitions of asymptotes.
  1. The line $x = a$ is a **vertical asymptote** of the graph of f if $f(x) \to \pm\infty$ as $x \to a$, either from the right or from the left.
  2. The line $y = b$ is a **horizontal asymptote** of the graph of f if $f(x) \to b$ as $x \to \pm\infty$.
- State the following **Rules for Asymptotes of Rational Functions.**
  Let f be a rational function given by

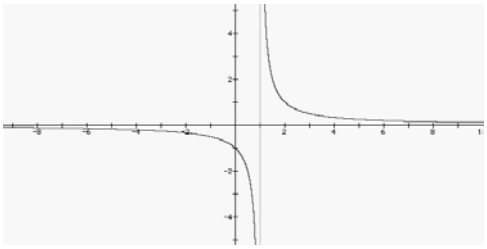
$$f(x) = \frac{N(x)}{D(x)} = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_n x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0}.$$

1. The graph of f has a *vertical* asymptote at $x = a$ if $D(a) = 0$ and $N(a) \neq 0$.
2. The graph of f has one *horizontal* asymptote or no horizontal asymptote, depending on the degree of N and D.
   a. If $n < m$, then $y = 0$ is the horizontal asymptote of the graph of f.
   b. If $n = m$, then $y = a_n/b_m$ is the horizontal asymptote of the graph of f.
   c. If $n > m$, then there is no horizontal asymptote of the graph of f.

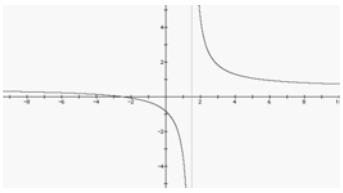
Example 2. Find any horizontal and vertical asymptotes of the following.

a) $f(x) = \dfrac{x+1}{x^2-1}$

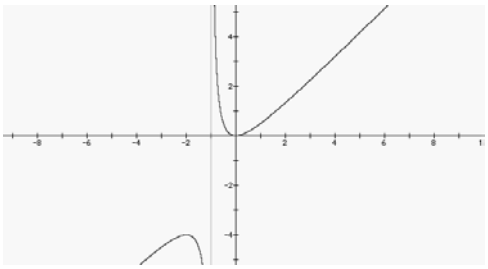
The horizontal asymptote is $y = 0$. The only vertical asymptote is $x = 1$. There will be a hole in the graph at $x = -1$.



b) $g(x) = \dfrac{2x+5}{4x-6}$. The horizontal asymptote is at $y = 1/2$, and the vertical asymptote is at $x = 3/2$.



c) $h(x) = \dfrac{x^2}{x+1}$. No horizontal asymptote and a vertical asymptote at $x = -1$.

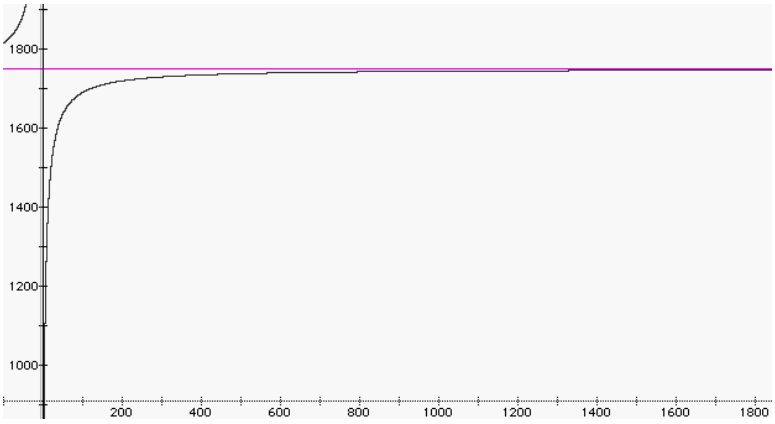


III. Applications (p. 335-336) Pace: 5 minutes

Example 3. A game commission has determined that if 500 deer are introduced into a preserve, the population at any time t (in months) is given by

$$N = \frac{500 + 350t}{1 + 0.2t}.$$

What is the carrying capacity of the preserve?



The carrying capacity will be equal to the y-value of the horizontal asymptote, $y = 1750$.