Chapter 1 Equations and Inequalities

Course/Section
Lesson Number
Date

Section 1.6 Other Types of Equations

Section Objectives: Students will know how to solve polynomial equations, radical equations, fractional equations, and absolute value equations.

I. Polynomial Equations (pp. 130-131) Pace: 10 minutes

- To start this section, we look at polynomial equations of higher degree. Our main tool will be the Zero-Factor Property.

Example 1. Solve.

a)

$$x^3 = 9x$$
$$x^3 - 9x = 0$$
$$x(x^2 - 9) = 0$$
$$x(x+3)(x-3) = 0$$
$$x = 0$$
$$x + 3 = 0 \Rightarrow x = -3$$
$$x - 3 = 0 \Rightarrow x = 3$$

Tip: Remind your students not to divide by x; they will lose a solution that way.

b)

$$x^3 - x^2 - 4x + 4 = 0$$
$$x^2(x-1) - 4(x-1) = 0$$
$$(x^2 - 4)(x-1) = 0$$
$$(x+2)(x-2)(x-1) = 0$$
$$x + 2 = 0 \Rightarrow x = -2$$
$$x - 2 = 0 \Rightarrow x = 2$$
$$x - 1 = 0 \Rightarrow x = 1$$

c)

$$x^4 - 9x^2 + 20 = 0$$
$$(x^2 - 4)(x^2 - 5) = 0$$
$$(x+2)(x-2)(x^2 - 5) = 0$$
$$x + 2 = 0 \Rightarrow x = -2$$
$$x - 2 = 0 \Rightarrow x = 2$$
$$x^2 - 5 = 0 \Rightarrow x = \pm\sqrt{5}$$

Tip: If students have trouble with the above factoring, make the substitution $u = x^2$ and then solve the equation. Be aware that students tend to forget to substitute back in for x.

II. Equations Involving Radicals (p. 132) Pace: 10 minutes

Tip: Students should be warned that the following methods can produce extraneous roots. Therefore, all alleged solutions should be checked in the original equation.

Example 2. Solve

a)

$$x^{3/2} - 27 = 0$$
$$x^{3/2} = 27$$
$$x = 27^{2/3} = 9$$

b)

$$\sqrt{x-3} + 5 = 0$$
$$\sqrt{x-3} = -5$$
$$\sqrt{x-3}^{\,2} = (-5)^2$$
$$x - 3 = 25$$
$$x = 28$$

By checking 28 in the original equation, we can see that there are no solutions.

c)

$$\sqrt{x-5} + \sqrt{x+7} = 6$$
$$\sqrt{x-5} = 6 - \sqrt{x+7}$$
$$\sqrt{x-5}^{\,2} = \left(6 - \sqrt{x+7}\right)^2$$
$$x - 5 = 36 - 12\sqrt{x+7} + (x+7)$$
$$-48 = -12\sqrt{x+7}$$
$$4^2 = \sqrt{x+7}^{\,2}$$
$$16 = x + 7$$
$$9 = x$$

This value checks in the original equation.

III. Equations with Fractions or Absolute Values (pp. 133-134)

Pace: 5 minutes

- Remind the students that the first step in solving equations that contain fractions is to multiply both sides by the LCD.

Example 3. Solve

$$\frac{x}{x-1} - \frac{6}{x} = 2$$
$$x(x-1)\left(\frac{x}{x-1} - \frac{6}{x}\right) = 2x(x-1)$$
$$x^2 - 6(x-1) = 2x^2 - 2x$$
$$x^2 + 4x = 6$$
$$x^2 + 4x + 4 = 6 + 4$$
$$(x+2)^2 = 10$$
$$x + 2 = \pm\sqrt{10}$$
$$x = -2 \pm \sqrt{10}$$

- To solve an equation involving absolute value, we rewrite it equivalently as two separate equations. For instance,
  $|x + 4| = 5 \Leftrightarrow x + 4 = 5$ and $x + 4 = -5$.

Example 4. Solve

$$|x^2 - 6| = x$$

$$x^2 - 6 = x$$
$$x^2 - x - 6 = 0$$
$$(x-3)(x+2) = 0$$
$$x - 3 = 0 \Rightarrow x = 3$$
$$x + 2 = 0 \Rightarrow x = -2$$

and

$$x^2 - 6 = -x$$
$$x^2 + x - 6 = 0$$
$$(x+3)(x-2) = 0$$
$$x + 3 = 0 \Rightarrow x = -3$$
$$x - 2 = 0 \Rightarrow x = 2$$

The solutions are ±2 and ±3.

IV. **Applications** (pp. 135-136) Pace: 10 minutes

Example 5. A train travels between two cities that are 300 miles apart. If the average speed of the train were increased by 10 mph, the travel time would decrease by one hour. What is the present average speed of the train?

Verbal Model: New Time = Present Time – 1

Labels: Present Time = 300/*r*
New Time = 300/(*r* + 10)

Equation: $\frac{300}{r+10} = \frac{300}{r} - 1$

$$r(r+10)\frac{300}{r+10} = r(r+10)\left(\frac{300}{r} - 1\right)$$
$$300r = 300(r+10) - r(r+10)$$
$$300r = 300r + 3000 - r^2 - 10r$$
$$r^2 + 10r - 3000 = 0$$
$$(r+60)(r-50) = 0$$
$$r + 60 = 0 \Rightarrow r = -60$$
$$r - 50 = 0 \Rightarrow r = 50$$

The present average rate of the train is 50 mph.

Example 6. Bob takes 10 minutes longer than Nancy to sweep the driveway. Together they can sweep the driveway in 12 minutes. How long does it take Bob to sweep the driveway working alone?

Verbal Model: Bob's rate + Nancy's rate = Rate for both

Labels: Bob's rate = $1/t$
Nancy's rate = 1/(t – 10)
Rate for both = 1/12

Equation: $\frac{1}{t} + \frac{1}{t-10} = \frac{1}{12}$

$$12t(t-10)\left(\frac{1}{t} + \frac{1}{t-10}\right) = \frac{1}{12} \cdot 12t(t-10)$$

$$12(t-10) + 12t = t(t-10)$$

$$24t - 120 = t^2 - 10t$$

$$t^2 - 34t + 120 = 0$$

$$(t-4)(t-30) = 0$$

$$t - 4 = 0 \Rightarrow t = 4$$

$$t - 30 = 0 \Rightarrow t = 30$$

Bob takes 30 minutes to sweep the driveway.

Example 7. If you deposit \$100 in an investment that is compounded monthly, the investment will be worth \$164.70 in 10 years. What is the interest rate?

Formula: $A = P(1 + r/n)^{nt}$

Labels: $A = 164.70$
$P = 100$
$n = 12$
$t = 10$
$r = ?$

Equation:

$$164.70 = 100\left(1 + \frac{r}{12}\right)^{12 \cdot 10}$$

$$1.6470 = \left(1 + \frac{r}{12}\right)^{120}$$

$$\sqrt[120]{1.6470} = \sqrt[120]{\left(1 + \frac{r}{12}\right)^{120}}$$

$$\sqrt[120]{1.6470} = 1 + \frac{r}{12}$$

$$\sqrt[120]{1.6470} - 1 = \frac{r}{12}$$

$$12\left(\sqrt[120]{1.6470} - 1\right) = r$$

The interest rate is about 5%.