Chapter 1 Equations and Inequalities

Course/Section
Lesson Number
Date

Section 1.4 Quadratic Equations

Section Objectives: Students will know how to solve quadratic equations.

I. Factoring (p. 109) Pace: 5 minutes

- Define a **quadratic equation** to be any equation that can be written in the form
$$ax^2 + bx + c = 0$$
where a, b, and c are real numbers with $a \neq 0$.
- We will look at four different ways of solving quadratic equations in this section. The first is by factoring. This method uses the Zero-Factor Property, which states if $ab = 0$, then $a = 0$ or $b = 0$.

Example 1. Solve by factoring.

a)
$$x^2 + 7x + 12 = 0$$
$$(x+3)(x+4) = 0$$
$$x + 3 = 0 \Rightarrow x = -3$$
$$x + 4 = 0 \Rightarrow x = -4$$

b)
$$5x^2 - 10x = 0$$
$$5x(x-2) = 0$$
$$5x = 0 \Rightarrow x = 0$$
$$x - 2 = 0 \Rightarrow x = 2$$

Tip: Use a graphing utility to show that these solutions are x-intercepts.

II. Extracting Square Roots (p. 110) Pace: 5 minutes

- State that if $u^2 = d$, where $d > 0$ and u is an algebraic expression, then $u = \pm\sqrt{d}$. Solving equations using this fact is called **extracting square roots**. Note that this fact is true because
$$u^2 = d$$
$$u^2 - d = 0$$
$$(u+\sqrt{d})(u-\sqrt{d}) = 0$$
$$u + \sqrt{d} = 0 \Rightarrow u = -\sqrt{d}$$
$$u - \sqrt{d} = 0 \Rightarrow u = \sqrt{d}$$

Tip: Students have a tendency to forget the "±." You may have to go to great lengths to overcome this.

Example 2. Solve by extracting square roots.

a)
$$16x^2 = 25$$
$$x^2 = \frac{25}{16}$$
$$x = \pm\sqrt{\frac{25}{16}} = \pm\frac{5}{4}$$

b)

$$\begin{aligned}(2x-1)^2 &= 6\\ 2x-1 &= \pm\sqrt{6}\\ 2x &= 1\pm\sqrt{6}\\ x &= \frac{1\pm\sqrt{6}}{2}\end{aligned}$$

III. Completing the Square (p. 111) Pace: 10 minutes

- State the following fact: $x^2+bx+\left(\frac{b}{2}\right)^2=\left(x+\frac{b}{2}\right)^2$. Hence, if we have $x^2+bx=c$, add $(b/2)^2$ to both sides, rewrite the left side, and solve by extracting square roots. This method is called **completing the square**.

Example 3. Solve by completing the square.

a)

$$\begin{aligned}x^2+4x &= 5\\ x^2+4x+4 &= 5+4\\ (x+2)^2 &= 9\\ x+2 &= \pm 3\\ x &= -2\pm 3\end{aligned}$$

The solutions are –5 and 1.

b)

$$\begin{aligned}2x^2-3x+1 &= 0\\ 2x^2-3x &= -1\\ x^2-\frac{3}{2}x &= -\frac{1}{2}\\ x^2-\frac{3}{2}x+\left(\frac{3}{4}\right)^2 &= -\frac{1}{2}+\left(\frac{3}{4}\right)^2\\ \left(x-\frac{3}{4}\right)^2 &= \frac{1}{16}\\ x-\frac{3}{4} &= \pm\frac{1}{4}\\ x &= \frac{3}{4}\pm\frac{1}{4}\end{aligned}$$

The solutions are 1/2 and 1.

IV. The Quadratic Formula (pp. 112-113) Pace: 10 minutes

- Start with $ax^2 + bx + c = 0$, $a \neq 0$, and solve by completing the square to derive the **Quadratic Formula**.

$$ax^2 + bx + c = 0$$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2|a|}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Tip: Students may be curious about the absence of the absolute value symbol in the final form. You might tell them that it got replaced by $\pm a$, and that this $\pm$ is in the numerator.

- The radicand of the Quadratic Formula is called the **discriminant**. It can tell us how many solutions the equation has, as follows:

  $b^2 - 4ac > 0$ implies two real solutions.
  $b^2 - 4ac = 0$ implies one real solution.
  $b^2 - 4ac < 0$ implies no real solutions.

Example 4. Solve by using the Quadratic Formula.

$$3x^2 - x - 5 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-5)}}{2(3)}$$

$$x = \frac{1 \pm \sqrt{61}}{6}$$

V. Applications (pp. 114-117) Pace: 15 minutes

Example 5. A rectangular room is 5 feet longer than it is wide. The area of the room is 374 square feet. Find the dimensions of the room.

Verbal Model: Length • Width = Area

Labels: Width = w (feet)
Length = w +5 (feet)
Area = 374 (square feet)

Equation:

$$
\begin{aligned}
(w+5)w &= 374\\
w^2+5w-374 &= 0\\
(w+22)(w-17) &= 0\\
w+22 = 0 &\Rightarrow w = -22\\
w-17 = 0 &\Rightarrow w = 17
\end{aligned}
$$

The room is 17 ft. by 22 ft.

Example 6. A group rents a cabin for $300 for a weekend. If one more person goes along, the price per person is reduced by $10. How many people are in the original group?

Verbal Model: Original cost per person – 10 = New cost per person

Labels: Original cost per person = 300/x (dollars)
New cost per person = 300/(x + 1) (dollars)

Equation:

$$
\begin{aligned}
\frac{300}{x}-10 &= \frac{300}{x+1}\\
x(x+1)\left(\frac{300}{x}-10\right) &= x(x+1)\frac{300}{x+1}\\
300(x+1)-10x(x+1) &= 300x\\
300+290x-10x^2 &= 300x\\
300-10x-10x^2 &= 0\\
x^2+x-30 &= 0\\
(x+6)(x-5) &= 0\\
x &= 5
\end{aligned}
$$

Example 7. Ranger Station 1 is due north of you. Ranger Station 2 is due east of you and 10 miles farther away from you than Ranger Station 1. If the two stations are 50 miles apart, how far are you from Ranger Station 2?

Verbal Model: (Distance 1)^2 + (Distance 2)^2 = 50^2

Labels: Distance 1 = x (miles)
Distance 2 = x + 10 (miles)

Equation:

$$
\begin{aligned}
x^2+(x+10)^2 &= 50^2\\
2x^2+20x+100 &= 2500\\
x^2+10x &= 1200\\
x^2+10x+25 &= 1200+25\\
(x+5)^2 &= 1225\\
x+5 &= \pm 35\\
x &= -5\pm 35
\end{aligned}
$$

You are 30 + 10 = 40 miles from Station 2.