Chapter 1 Equations and Inequalities

Section 1.2 Linear Equations in One Variable

Course/Section
Lesson Number
Date

Section Objectives: Students will know how to solve linear equations and how to find x- and y-intercepts algebraically .

I. Equations and Solutions of Equations (p. 88) Pace:5 minutes

- An **equation** in x is a statement that two variable expressions are equal. A **solution** of an equation is a number r such that when x is replaced by r, the resulting equation is a true statement. The **solution set** of an equation is the set of all solutions of the equation. Two equations are said to be **equivalent** if they have the same solution set. To **solve** an equation means to find its solution set. We will encounter two types of equations. One is the identity, in which every real number in the domain of the variable is a solution. The other is the conditional equation, in which only some of the numbers in the domain of the variable are solutions. The latter is the type that we solve.

II. Linear Equations in One Variable (pp. 88-90) Pace:15 minutes

- A **linear equation in one variable** has a polynomial on each side of the equation, of degree no greater than one. In solving such equations we want to sequentially produce equivalent equations, each simpler than the previous, until the solution is obvious. Here are the four ways of producing equivalent equations.

  i) Replace any expression with an equivalent one, using the rules of algebra from the previous chapter.
  ii) Add (or subtract) the same quantity to (from) both sides of the equation.
  iii) Multiply (or divide) both sides of the equation by the same *nonzero* quantity.
  iv) Interchange the two sides of the equation.

  Example 1. Solve
  a)

$$\begin{aligned} 5x-7&=3\\ 5x&=10\\ x&=2 \end{aligned}$$

  b)

$$\begin{aligned} -2(x-1)&=5(3x-8)\\ -2x+2&=15x-40\\ -17x+2&=-40\\ -17x&=-42\\ x&=\frac{42}{17} \end{aligned}$$

- This is a good time to use a graphing calculator to show students where these solutions show up on the graph. Also, attention should be drawn to the Historical Note on page 89.

III. Equations That Lead to Linear Equations (p. 91)

Pace: 10 minutes

- If an equation contains fractions, a good first step in solving is to multiply both sides of the equation by the LCD.

Example 2. Solve

a)

$$\frac{x}{3}-\frac{2x}{5}=\frac{1}{6}$$

$$30\left(\frac{x}{3}-\frac{2x}{5}\right)=30\cdot\frac{1}{6}$$

$$10x-12x=5$$

$$-2x=5$$

$$x=-\frac{5}{2}$$

b)

$$\frac{x}{x-1}=\frac{1}{x-1}-\frac{1}{x-3}$$

$$(x-1)(x-3)\cdot\frac{x}{x-1}=(x-1)(x-3)\cdot\left(\frac{1}{x-1}-\frac{1}{x-3}\right)$$

$$x(x-3)=(x-3)-(x-1)$$

$$x^2-3x=-2$$

$$x^2-3x+2=0$$

$$(x-1)(x-2)=0$$

$$x=2$$

Note that $x = 1$ is an extraneous solution because it leads to division by zero.

IV. Finding Intercepts Algebraically (p. 92) Pace:5 minutes

- To find x-intercepts algebraically, set y equal to zero and solve the equation for x. To find y-intercepts algebraically, set x equal to zero and solve for y.

Example 3. Find the x- and y-intercepts of the graph of each equation.

a) $2x + 3y = 6$

To find the y-intercept, let $x = 0$ and solve for y.

$$2(0)+3y=6$$

$$3y=6$$

$$y=2$$

Hence (0, 2) is the y-intercept.

To find the x-intercept, let $y = 0$ and solve for x.

$$2x+3(0)=6$$

$$2x=6$$

$$x=3$$

Hence (3, 0) is the x-intercept.

b) $y = x^2 + x - 6$

To find the y-intercept, let $x = 0$ and solve for y.

$$y = 0^2 + 0 - 6 = -6$$

Hence (0, -6) is the y-intercept.

To find the x-intercept, let $y = 0$ and solve for x.

$$0 = x^2 + x - 6$$

$$0 = (x+3)(x-2)$$

$$x = -3 \text{ or } x = 2$$

Hence (-3, 0) and (2, 0) are the x-intercepts.

V. Application (p. 92) Pace:5 minutes

Example 4. A delivery company has determined that its average annual operating cost is

$C = 1.19x + 2100$

where x is the number of packages delivered per year.

(a) Find the y-intercept of the graph of the linear model algebraically.

Let $x = 0$ and solve for C.

$C = 1.19x + 2100$

$C = 1.19(0) + 2100$

$C = 2100$

The y-intercept is (0, 2100).

(b) How many packages can this company deliver in a year with an operating cost of $12,000?

$$12,000 = 1.19x + 2100$$

$$9900 = 1.19x$$

$$\frac{9900}{1.19} = x$$

The company can deliver approximately 9899 packages.