Chapter 9, Part B
Hypothesis Testing

- Population Proportion
- Hypothesis Testing and Decision Making
- Calculating the Probability of Type II Errors
- Determining the Sample Size for a Hypothesis Test About a Population Mean

A Summary of Forms for Null and Alternative Hypotheses About a Population Proportion

- The equality part of the hypotheses always appears in the null hypothesis.
- In general, a hypothesis test about the value of a population proportion p must take one of the following three forms (where p_0 is the hypothesized value of the population proportion).

$H_0: p \geq p_0$ $H_a: p < p_0$	$H_0: p \leq p_0$ $H_a: p > p_0$	$H_0: p = p_0$ $H_a: p \neq p_0$
One-tailed (lower tail)	One-tailed (upper tail)	Two-tailed

Tests About a Population Proportion

- Test Statistic

$$z = \frac{\bar{p} - p_0}{\sigma_{\bar{p}}}$$

where:

$$\sigma_{\bar{p}} = \sqrt{\frac{p_0(1-p_0)}{n}}$$

assuming $np \geq 5$ and $n(1 - p) \geq 5$

Tests About a Population Proportion

- Rejection Rule: p –Value Approach

Reject H_0 if p –value $\leq \alpha$

- Rejection Rule: Critical Value Approach
  - H_0: $p \leq p_0$ Reject H_0 if $z \geq z_\alpha$
  - H_0: $p \geq p_0$ Reject H_0 if $z \leq -z_\alpha$
  - H_0: $p = p_0$ Reject H_0 if $z \leq -z_{\alpha/2}$ or $z \geq z_{\alpha/2}$

Two-Tailed Test About a Population Proportion

- Example: National Safety Council (NSC)

For a Christmas and New Year's week, the National Safety Council estimated that 500 people would be killed and 25,000 injured on the nation's roads. The NSC claimed that 50% of the accidents would be caused by drunk driving.

A sample of 120 accidents showed that 67 were caused by drunk driving. Use these data to test the NSC's claim with α = .05.

Two-Tailed Test About a Population Proportion

- p –Value and Critical Value Approaches

1. Determine the hypotheses. $H_0: p = .5$ $H_a: p \neq .5$

2. Specify the level of significance. α = .05

3. Compute the value of the test statistic.

$$\sigma_{\bar{p}} = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{.5(1-.5)}{120}} = .045644$$

$$z = \frac{\bar{p} - p_0}{\sigma_{\bar{p}}} = \frac{(67/120) - .5}{.045644} = 1.28$$

a common error is using $\bar{p}$ in this formula

Two-Tailed Test About a Population Proportion

- *p*–Value Approach
  - 4. Compute the *p*-value.

    For z = 1.28, cumulative probability = .8997

    p–value = 2(1 – .8997) = .2006
  - 5. Determine whether to reject H_0.

    Because p–value = .2006 > α = .05, we cannot reject H_0.

Two-Tailed Test About a Population Proportion

- Critical Value Approach
  - 4. Determine the criticals value and rejection rule.

    For $\alpha/2$ = .05/2 = .025, $z_{.025}$ = 1.96

    Reject H_0 if $z \leq$ -1.96 or $z \geq$ 1.96
  - 5. Determine whether to reject H_0.

    Because 1.278 > -1.96 and < 1.96, we cannot reject H_0.

Hypothesis Testing and Decision Making

- Thus far, we have illustrated hypothesis testing applications referred to as significance tests.
- In the tests, we compared the *p*-value to a controlled probability of a Type I error, α, which is called the level of significance for the test.
- With a significance test, we control the probability of making the Type I error, but not the Type II error.
- We recommended the conclusion "do not reject H_0" rather than "accept H_0" because the latter puts us at risk of making a Type II error.

Hypothesis Testing and Decision Making

- With the conclusion "do not reject H_0", the statistical evidence is considered inconclusive.
- Usually this is an indication to postpone a decision until further research and testing is undertaken.
- In many decision-making situations the decision maker may want, and in some cases may be forced, to take action with both the conclusion "do not reject H_0" and the conclusion "reject H_0."
- In such situations, it is recommended that the hypothesis-testing procedure be extended to include consideration of making a Type II error.

Calculating the Probability of a Type II Error in Hypothesis Tests About a Population Mean

1. Formulate the null and alternative hypotheses.
2. Use the level of significance α and the critical value approach to determine the critical value and the rejection rule for the test.
3. Use the rejection rule to solve for the value of the sample mean corresponding to the critical value of the test statistic.

Calculating the Probability of a Type II Error in Hypothesis Tests About a Population Mean

4. Use the results from step 3 to state the values of the sample mean that lead to the acceptance of H_0; this defines the acceptance region.
5. Using the sampling distribution of $\bar{x}$ for a value of μ satisfying the alternative hypothesis, and the acceptance region from step 4, compute the probability that the sample mean will be in the acceptance region. (This is the probability of making a Type II error at the chosen level of μ .)

Calculating the Probability of a Type II Error

- Example: Metro EMS (revisited)

Recall that the response times for a random sample of 40 medical emergencies were tabulated. The sample mean is 13.25 minutes. The population standard deviation is believed to be 3.2 minutes.

The EMS director wants to perform a hypothesis test, with a .05 level of significance, to determine whether or not the service goal of 12 minutes or less is being achieved.

Calculating the Probability of a Type II Error

1. Hypotheses are: H_0: $\mu \leq 12$ and H_a: $\mu > 12$
2. Rejection rule is: Reject H_0 if $z \geq 1.645$
3. Value of the sample mean that identifies the rejection region:

$$z = \frac{\bar{x} - 12}{3.2/\sqrt{40}} \geq 1.645$$

$$\bar{x} \geq 12 + 1.645\left(\frac{3.2}{\sqrt{40}}\right) = 12.8323$$

4. We will accept H_0 when $\bar{x} < 12.8323$

Calculating the Probability of a Type II Error

5. Probabilities that the sample mean will be in the acceptance region:

Values of μ	$z = \frac{12.8323 - \mu}{3.2/\sqrt{40}}$	β	1-β
14.0	-2.31	.0104	.9896
13.6	-1.52	.0643	.9357
13.2	-0.73	.2327	.7673
12.8323	0.00	.5000	.5000
12.8	0.06	.5239	.4761
12.4	0.85	.8023	.1977
12.0001	1.645	.9500	.0500

Calculating the Probability of a Type II Error

- Calculating the Probability of a Type II Error

Observations about the preceding table:

- When the true population mean μ is close to the null hypothesis value of 12, there is a high probability that we will make a Type II error.

Example: μ = 12.0001, β = .9500

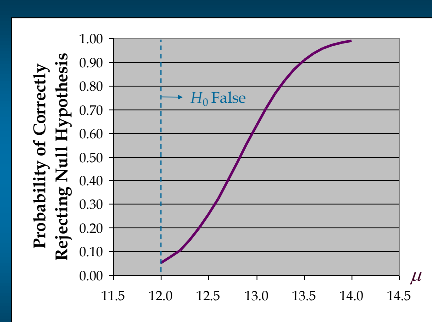
- When the true population mean μ is far above the null hypothesis value of 12, there is a low probability that we will make a Type II error.

Example: μ = 14.0, β = .0104

Power of the Test

- The probability of correctly rejecting H_0 when it is false is called the power of the test.
- For any particular value of μ, the power is 1 – β.
- We can show graphically the power associated with each value of μ; such a graph is called a power curve. (See next slide.)

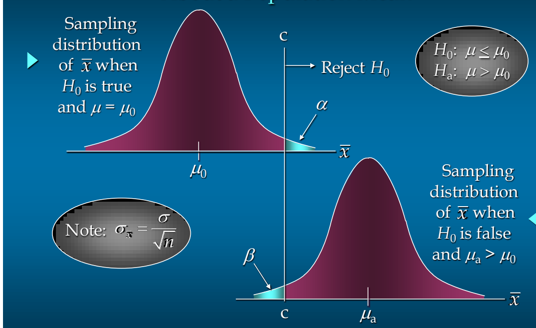
Power Curve



Determining the Sample Size for a Hypothesis Test About a Population Mean

- The specified level of significance determines the probability of making a Type I error.
- By controlling the sample size, the probability of making a Type II error is controlled.

Determining the Sample Size for a Hypothesis Test About a Population Mean



Determining the Sample Size for a Hypothesis Test About a Population Mean

$$n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{(\mu_0 - \mu_a)^2}$$

where

z_α = z value providing an area of α in the tail
z_β = z value providing an area of β in the tail
σ = population standard deviation
μ_0 = value of the population mean in H_0
μ_a = value of the population mean used for the Type II error

Note: In a two-tailed hypothesis test, use $z_{\alpha/2}$ not z_α

Determining the Sample Size for a Hypothesis Test About a Population Mean

- Let's assume that the director of medical services makes the following statements about the allowable probabilities for the Type I and Type II errors:
  - •If the mean response time is μ = 12 minutes, I am willing to risk an α = .05 probability of rejecting H_0.
  - •If the mean response time is 0.75 minutes over the specification (μ = 12.75), I am willing to risk a β = .10 probability of not rejecting H_0.

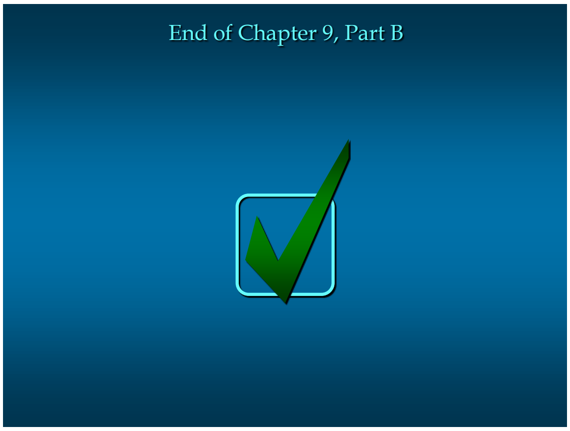
Determining the Sample Size for a Hypothesis Test About a Population Mean

α = .05, β = .10
z_α = 1.645, z_β = 1.28
μ_0 = 12, μ_a = 12.75
σ = 3.2

$$n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{(\mu_0 - \mu_a)^2} = \frac{(1.645 + 1.28)^2 (3.2)^2}{(12 - 12.75)^2} = 155.75 \approx 156$$

Relationship Among α, β, and n

- Once two of the three values are known, the other can be computed.
- For a given level of significance α, increasing the sample size n will reduce β.
- For a given sample size n, decreasing α will increase β, whereas increasing α will decrease b.


End of Chapter 9, Part B