Propagating measurement uncertainty via binomial expansions

A box of Yardley London Flowering English Lavender soap is 9.6 cm long by 3.9 cm thick by 6.1 cm wide. The uncertainty in each edge is a half a millimeter, or 0.05 cm. Calculate the volume of the box and include the uncertainty.

Thus the longest edge is 9.6 cm plus or minus 0.05 cm and so forth. The volume is just the length × width × height

(**9.6** ± 0.05) (**3.9** ± 0.05) (**6.1** ± 0.05)

(**37.44** ± 0.48 ± 0.195 ± 0.0025) (**6.1** ± 0.05)

228.384 ± 1.872 ± 0.01525 ± 1.1895 ± 2.928 ± 0.000125 ± 0.00975 ± 0.024

228.384 ± 6.038625

Notice that the 0.05 centimeter uncertainty in any one edge is now a **SIX** cubic centimeter uncertainty in the volume of the soap box.

That means the actual soap box volume could be anything from 222.345375 cm^3 to 234.422625 cm^3 !! Never mind the decimal places – we are uncertain in the ONES place: the volume is between 222 and 234 cm^3 So writing down ANY decimal place is SILLY. They have NO MEANING.

A scientist would use the above type of analysis to write the volume of the soap box as:

228 ± 6 cm^3

Notice that although the initial measurement is to one decimal place, the volume has no decimal places.

This approach is usually mathematically beyond an SC 130 physical science student. The simpler approach that yields answers close to the mathematically correct answer is to count the significant digits – the non-zero digits in the measurement and then keep that number of digits in the answer. The word digits means "numbers." Each measure has two significant digits (some people use the term significant figures). So the answer gets to have only two significant digits (non-zero digits):

①②
2 3 0 cm^3

Note that while 230 cm^3 differs from the answer of the scientist, it is between 222 and 234 cm^3 This is an acceptable measure of the volume, with the zero telling the trained reader that the last digit is uncertain. Just as noted by the scientist: ± 6 says the 8 is uncertain.

So count your digits!